A Model of Artists' Time Allocation

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Abstract

For centuries, it was necessary for performers to be present in 'real time' in order to supply their services, such as music, dance or drama. Labour time and human skill and capital were inextricably related. 'Reproducibility' – the ability to make copies of human services that are adequate substitutes for 'live' performance – has meant performers need not be present to supply their services, which can be recorded and supplied with economies of scale. On the other hand, consumers do not need the live performer to be present in order to obtain her services. Demand for performers has therefore fallen. However, those remaining in the market have the choice of whether to work 'live' for 'spot' fees or to allocate their time to making recordings which will earn them income through royalties and repeat fees over the duration of the copyright of the performance. The paper considers this situation in terms of a formal model of time allocation.

1. Introduction

The evolution of copyright law is inextricably connected to the ability to make mechanical (and now digital) copies that began with the invention of the printing press. The development of recording technologies – sound recording, motion picture making, photocopiers, home recording equipment (VCRs, CD burners) and the internet – that duplicate a work from a master copy (a performance, a book, a photograph) has vastly extended reproducibility. These inventions have had a fundamental effect on the way consumers access the arts and other cultural products and have vastly altered the labour market for creative artists and performers.

The exploitation of work embodied in reproducible form has a double-sided effect: it 'alienates' the author's creative input from her labour, as the work can now reach the market without the necessity of her presence; and, through copyright law, the publisher acquires a durable asset, which he can exploit independently of the author (who may even be dead since the copyright term is life plus 70 years[1]).

One effect of the combination of copyright law and 'reproducibility' on artists' labour markets that does not seem to have been explored in the literatures of either the economics of copyright or cultural economics is how this combination affects artists' supply decisions. For centuries, it was necessary for performers to be present in 'real time' in order to supply their services, such as music, dance or drama. Labour time and human skill and capital were inextricably related. 'Reproducibility' – the ability to make copies of human services that are adequate substitutes for 'live' performance – has meant performers need not be present to supply their services, which can be recorded and supplied in conditions in which there are economies of scale in the use of artists' time. And as consumers no longer needed the live performer to be present in order to obtain her services, demand for performers fell dramatically over the 20th century. However, the durability of the recorded asset, protected by copyright, enables the performer to earn royalties over the life of the work.

Due to these features, the performer faces a labour supply decision that involves allocation decisions over time. She can decide to allocate her time to earn a spot price now or a future return – say a choice between doing a sound recording in preference to a concert; the concert pays a fee and the sound recording a royalty. Moreover, royalties keep coming in while the artist is occupied with other work, so the artist can earn more cumulatively in a given period of time. In addition to dividing their time between arts and non-arts work (a feature of artists' labour markets that has been well established – see, for example, Towse, 2001), artists can optimise a portfolio of copyrights that form part of an inter-temporal decision about present and future earnings. Taking this into account, and ignoring the opportunity cost of the non-arts wage, an artist's earnings at any point in time depend upon wages and fees for the hours of work done in that period plus copyright royalty income (the royalty rate times the number of copyrights the artist holds). It is to be expected that the higher is royalty income, the fewer hours of work artists would do in any given period. A

[1] It is often forgotten that an author's work is protected for a longer period than the copyright term. Say an author creates a work at the age of 25 and she lives to the age of 75, that work is protected for 120 years. However, performers' work is protected for 50 years from the date of fixation. A singer-songwriter has the authors' copyright and the performers' neighbouring rights.

model along these lines could be tested using data from artists' surveys that asked for separate information on fees and wages and for royalties.

It is not difficult to see that the choice faced by an artist about how to best spend her time between the two activities is very closely related to a model of financial portfolio theory in which there exist two assets – a short term certain return asset (akin to the 'live performance' option of the artist), and a long term perpetuity asset (akin to the 'copyright' activity of our artist). There are, however, some differences that we can at least mention here, although full and formal consideration of them will make the model that follows too complex.

Firstly, for the case of the artist, the choice of what to do is really more complex than is the financial portfolio choice. The artist is considering how to spend her time, and so there is always the third option of leisure. In the financial choice counterpart, 'leisure' would be similar to holding money in a non-interest rate generating account[2], something that is normally assumed to be strongly dominated by the certain return asset. Thus the 'leisure' equivalent in the finance model is normally not included, and the problem there is often cast as that of sharing wealth over two options, one risky and one safe. We know, however, from the very basics of undergraduate economics, that the choice between leisure and paid employment is not a simple one, and the interplay between wealth and substitution effects may imply that the supply of labour is backward bending. In what follows, we shall make a very rudimentary attempt to consider leisure in a first model (in section 2), but in our principle model (section 3) we must practically assume away the leisure choice. Their, we must assume that either leisure is held constant, i.e. it is not a choice variable, or at best that it is an activity that is held proportionately constant with all activities that do not generate royalty income. Otherwise there are simply too many variables to take into account for a non-complex analysis. Clearly a better treatment of leisure is a very obvious extension that should be considered in the future.[3]

Secondly, we could also allow the artist the option of saving (either positive or negative) from one period to the next. Doing so would again increase the dimensionality of the choice, and thus complicate things excessively. However, we

[2] Such an account may still generate utility, primarily due to the consumption opportunities that it opens up.

[3] Throsby (1994) has suggested a work preference model of artists' labour supply, in which the artist gains utility from devoting time to her art.

will be able to come to some sweeping statements about how wealth accumulation over time may affect the time allocation problem itself.

Finally, there are clear externalities from one activity to the other that we will not take into account formally. Specifically, one would expect that the perceived quality of the output of each activity would affect the payoffs from the other. For example, an artist with a very successful stage show can expect to generate a larger fan base, and is thereby likely to earn more on record sales (copyright income). The opposite is also true – popular artists in record sales terms are likely to earn more revenues from live concerts. We shall mention these externalities from time to time, but again they are just too complex to introduce in a formal way into the analysis.

On the other hand, in a financial portfolio setting, the investor at any period may decide to either add to or subtract from the investment in the perpetuity asset. In the setting considered here, that would imply that the artist is able to either add to or subtract from her copyright portfolio. Now clearly it is simple to add to it, by recording new material and making that material available for distribution, but subtracting from the copyright portfolio may be more difficult. There are some rights included under copyright that simply cannot be traded away (e.g. moral rights), although in principle the artist could sell off part of her copyright portfolio in the market. The problem here is, of course, that it is not really true that all elements in the copyright portfolio are equal, and so the choice of which elements to retain and which to sell is complex. We avoid the problem entirely, by only considering additions to the copyright portfolio, and abstracting away entirely from the option of selling off parts of this portfolio.

2. A Single Period Model With Leisure

To start off with, let us assume a very simplistic setting that only takes into account one period at a time. As we shall see, however, even this setting allows us to see at least one important aspect of the inter-temporal choice of the artist between work and leisure.

The artist has T units of time at her disposal, which is normalised to 1, and she must decide how to split this time between three activities – paid 'spot' employment (which we shall denominate 'live performances'), royalty generating activities (that is, ‘recording’)[4], and leisure. Let the amount of leisure be indicated by L, so that once leisure has been decided, the artist has $1-L$ units of time to dedicate to wealth generating activities. Assume then, that she decides to spend a proportion a of this non-leisure time working on live performing, and a proportion $1-a$ on the royalty generating activity. Assume that each unit of time spent in live performances yields a wage payment of w, and that each unit of time spent in recording yields a payment of r. The term ‘royalty’ is used here somewhat loosely, since there is only one period assumed. We can, however, associate this situation with one in which in each period the artist sells her copyright in the royalty generating output outright to a second party (perhaps a record label), in which case r is the price at which this transaction is realised, presumably equal to the present value of the royalty income stream that accrues to the new owner of the copyright, divided by the amount of time that the artist dedicated to the royalty generating activity. Thus the artist earns $wa(1-L)$ from live performances and $r(1-a)(1-L)$ from recording. We also assume that the artist has wealth of W independently of her current period’s activities, perhaps accumulated savings from earlier periods. Finally, we assume that the artist’s utility function for money, x, and leisure is $u(x,L)$, which is assumed to be strictly increasing in both arguments ($u_i'(x,L)>0 \; i=x,L$), strictly concave in both arguments ($u_{ii}''(x,L)<0 \; i=x,L$), and we assume a non-negative cross derivative ($u_{ij}''(x,L)\geq 0 \; i,j=x,L$).[5]

Clearly, the artist’s total wealth in the period under consideration is $x=(wa+r(1-a))(1-L)+W$, and her problem is to choose (x,L) so as to maximise $u(x,L)$, subject to $0\leq a\leq 1, 0\leq L\leq 1$. This is a relatively straightforward maximisation problem, with a strictly convex feasible set[6] and a strictly quasi-concave objective function. It can be quite simply solved using standard maximisation techniques (e.g. Lagrange), but here we shall simply point out some of the more salient features of the solution.

[4] These terms are used losely. For example, “recording” should include any activity that contributes to the generation of royalties; i.e. not only actual recording activity itself but also writing music and songs, etc. On the other hand, “live performances” should really include anything that is not related to generating royalties, including employment such as music teaching, or even employment outside of the business of music entirely.

[5] A non-negative cross derivative is sufficient for utility to be strictly quasi-concave in the vector (x,L), that is, for indifference curves in the space (x,L) to be convex.

Firstly, note that the first derivative of the objective function with respect to *a* is

$$u_x'(x,L)(w-r)(1-L)$$

Assuming that some positive amount of leisure is chosen, this is positive, negative, or equal to 0 as *w* is greater than, less than, or equal to *r*. If for example $w > r$, then the marginal utility of an additional unit of time spent in live performances is always positive (and consequently, the marginal utility of an additional unit of time spent in recording is always negative). Thus the optimum in this case is to set $a^* = 1$. Similarly, if $w < r$ then the optimum is $a^* = 0$, and if $w = r$ there is no uniquely optimal value of *a*. So, outside of the case in which the two wages are equal, there is always a corner solution for the value of *a*.

Secondly, the first derivative of the objective function with respect to *L* is

$$-u_x'(x,L)(wa + r(1-a)) + u_L'(x,L)$$

which must equal 0 if indeed there is to be an internal optimum in leisure. Thus the first order condition for leisure reads

$$\frac{u_x'(x,L)}{u_L'(x,L)} = \frac{1}{wa + r(1-a)}$$

But since we know that in any unique solution *a* is either 0 or 1 depending on the relationship between *w* and *r*, the right-hand-side of this is either 1/*w* or 1/*r*.

Let us assume, for example, that *r* is greater than *w*, so that the artist spends all of her non-leisure time recording ($a^* = 0$) in which case the first order condition for leisure reads

$$\frac{u_x'(x,L)}{u_L'(x,L)} = \frac{1}{r}$$

where now we have $x = r(1-L) + W$.

It is easy to see that under the assumptions on the derivatives that we have made, in this model leisure is a normal good, in the sense that an increase in *W* will lead to an increase in the optimal value of *L*. To see why, simply note that

$$\frac{\partial}{\partial W} u_x'(x,L) = u_{xx}''(x,L) < 0$$

$$\frac{\partial}{\partial W} u_L'(x,L) = u_{Lx}''(x,L) \geq 0$$

[6] The feasible set, in the space of the choice variables (*a*,*L*) is the unit square.

Thus, an increase in W will certainly decrease the value of the left-hand-side of our equilibrium condition

$$\frac{u'_x(x,L)}{u'_L(x,L)} = \frac{1}{r}$$

Since the right-hand-side of the condition is not altered, the artist must adjust L such that the left-hand-side is increased back to its original position. But since

$$\frac{\partial}{\partial L} u'_x(x,L) = -ru''_{xx}(x,L) + u''_{xL}(x,L) > 0$$

$$\frac{\partial}{\partial L} u'_L(x,L) = -ru''_{Lx}(x,L) + u''_{LL}(x,L) < 0$$

it turns out that the only way this can be done is by increasing L which will have the desired effect of increasing the right-hand-side of the equilibrium condition back to its equilibrium level. Thus an increase in accumulated wealth, W, will have the effect of increasing the amount of leisure taken.

The fact that leisure is necessarily a normal good here implies that we will not be able to sign the effect of an increase in the wage rate of the chosen activity upon L, simply because the wealth and the substitution effects of the change will have opposing effects on the demand for leisure. An increase in w cannot affect the solution (so long as both before and after the increase in w we still have r greater than w). However, the important point that we arrive at in this model is that, assuming homogeneous (and time independent) preferences, the more wealth that accrues over time in the form of inter-temporal savings, the more leisure will be demanded by the artist.

3. A "Simple" Model of Inter-temporal Time Allocation for Artists

The model of the previous section is overly simplistic for several reasons. But perhaps the most blatant is the single period assumption – that is, the assumption that in each period in which future royalty income is generated, the artist sells the rights to the royalty income stream to a second party. In this section we attempt to correct for this by developing a model in which the artist retains the future royalty stream, as is typically the case in the real-world. The model attempts to analyse the choice of how time is allocated between the activities of recording (the royalty generating activity) and live performing (the non-royalty generating activity), in any given period, when the effects of the choice in that period is felt in each of the future periods through the royalty income that accrues over time.

Our basic assumptions are very similar to the previous model. Assume an artist who can work in two different activities – activity 1 generates a per-unit-time wage of *w*, but no future income, and activity 2 generates a per-unit-time wage of *r* for the present period and an income stream to be received in each subsequent period. Thus, activity 2 is a royalty generating activity. There is nothing at all inconsistent with assuming that a part of the "non-royalty generating activity" is leisure, and the relevant wage rate, *w*, reflects the individual monetary equivalent of such activity. That is, activity 1 can be thought of as a composite activity, divided somehow between non-royalty paid employment (including live performances) and leisure activities. The composite wage rate *w* reflects the consumption value to the individual of a unit of time spent in activity 1, both money earned in paid employment and then spent on consumption and the consumption value of directly consuming leisure. This interpretation leaves leisure present in the model, at least to a certain degree, but since *w* is held constant the proportion of the non-royalty activity that is to be represented by leisure is held constant – an application of the "composite commodity theorem".

We assume that royalty payments are themselves discounted over time, by a factor *p*, where $0 \le p \le 1$. That is, for a recording created in period *i*, the royalty that is generated in period $i+j$ is discounted by p^j, so in the same period in which the recording is made ($j=0$), the royalty payment is not discounted, it is discounted by *p* in the next period, and so forth. This discounting assumption can be interpreted as the logical scenario in which the royalty earnings corresponding to a given recording depreciate over time. It can also be interpreted as a rudimentary attempt to model the fact that even though the copyright in a work may last a long time, there is a certain likelihood (here *p* per period) that the economic good that 'delivers' the copyrighted work is deleted from the catalogue by the publisher and thus no longer capable of earning the royalty; decision rights over copyrights typically are transferred to the publisher or record label (Caves, 2000). The degree to which the royalty is depreciated will depend, among other things, upon the strength of copyright protection that is available. A special case emerges when $p=1$ which is when the royalty income is a guaranteed sum in all future periods.

The artist has a per-period utility function for money, $u(x)$, that is assumed to be strictly increasing and concave. The artist is not allowed to either save money from period to period, nor can she loan money. Assume that the artist is endowed with a total amount of time per period of *T*, which again we normalise to 1. In this model, we do not allow the artist to also choose leisure. Leisure is implicitly set at 0, or alternatively the total time available in any period is $T+L=1+L$, and L is not a choice variable.

In each period the artist decides a proportion of her available time, say *a*, to dedicate to live performing, leaving a proportion $1-a$ for recording. We assume that the per-period discount factor is *d*, which is of course strictly positive and less than 1, and we assume exponential discounting.

To see how the model works, consider the first few periods. In period 1, the artist chooses a_1 such that her income in period 1 is $a_1 w+(1-a_1)r$. In period 2, her choice of a_2 gives her an income of $a_2 w+(1-a_2)r+(1-a_1)pr$. In general, in period *k*, the choice of a_k yields an income during that period of

$$a_k w + r\sum_{i=1}^{k} p^{k-i}(1-a_i) \equiv c_k \qquad (1)$$

In that case, assuming that the artist is infinitely lived, her problem is to maximise

$$J(a) \equiv \sum_{i=1}^{\infty} d^{i-1}u(c_i)$$

with respect to the vector $a=(a_1,a_2,...)$, where the only restriction on the choice is that $0 \le a_i \le 1$ for all *i*.

The second order condition for an optimal solution is fulfilled by the assumption that the utility function is concave, and so the first order conditions for an optimal solution are given by setting the first derivative of $J(a)$ with respect to each a_k equal to 0. Carrying out the implied derivative, we get

$$\frac{\partial J(a)}{\partial a_k} = d^{k-1}u'(c_k^*)\frac{\partial c_k}{\partial a_k} + \sum_{i=k+1}^{\infty} d^{i-1}u'(c_i^*)\frac{\partial c_i^*}{\partial a_k} = 0 \qquad k=1,2,....$$

But, from (1) we have

$$\frac{\partial c_k}{\partial a_k} = w-r \qquad \left.\frac{\partial c_i^*}{\partial a_k}\right|_{i>k} = -rp^{i-k}$$

Therefore the first order conditions are:

$$d^{k-1}u'(c_k^*)(w-r)-r\sum_{i=k+1}^{\infty}d^{i-1}p^{i-k}u'(c_i^*)=0 \qquad k=1,2,....$$

But dividing by d^{k-1} yields

$$u'(c_k^*)(w-r)-r\sum_{i=k+1}^{\infty}(dp)^{i-k}u'(c_i^*)=0 \qquad k=1,2,.... \tag{2}$$

And finally, since $(dp)^{k-k}=1$, we can write the first order conditions more simply as

$$wu'(c_k^*)-r\sum_{i=k}^{\infty}(dp)^{i-k}u'(c_i^*)=0 \qquad k=1,2,.... \tag{3}$$

The first term on the left-hand-side of (3) can be interpreted as the marginal revenue from increasing a_k and the second term on the left-hand-side can be thought of as the marginal cost of such an increase.

Clearly, from (2) for an internal solution to the problem, we require $w>r$. The reason is obvious; if the per-unit-time wage for the royalty generating acitivty were greater than that of activity 1, then a marginal increase in a_k (that is, dedicating a marginally greater amount of time to activity 1) has a negative impact on income in period *k*, and in all future periods as well. Thus in such a setting it would be optimal to dedicate all of the available time to royalty generation, i.e. the optimal solution would be to set $a_k^*=0$ for all *k*. Given this, from now on we explicitly assume $w>r$.

We can also note that the system of equations that defines the optimal vector is inter-temporally consistent, in the sense that the entire vector can be chosen at the outset (in period 1) without any of its components ever being altered as time goes by (so long as none of the system's parameters change). This is entirely due to the assumption of complete certainty; in an uncertain environment as information is revealed over time, the optimal vector is altered over time.

The fact that we are assuming an infinitely lived artist is not innocuous. If the artist did have a finite life span, then there exists a simple intuition as to the sequence of optimal time allocations. Clearly, in the final period of life no dedication to the generation of royalties is ever optimal, so in that period we get $a=1$. Working backwards, assuming that at some point sufficiently before the date of extinction, some royalty generation is done (that is assuming we do not have a boundary solution in which the artist never participates in activity 2 in any period), then it seems logical that over time less and less allocation will be given to generating royalties, and more and more to other activities. This is simply because the royalty activity provides for future income, as well as current income, so it seems reasonable that when the artist is "young" she will be more willing to spend her time in such an activity, since she has more periods during which to enjoy the stream of royalty income. On the other hand, when old, the artist will likely decide to spend more time earning the higher wage for non-royalty income, since the royalty income stream is likely to be short. Thus, it seems logical that the vector of optimal time dedication is increasing, in the sense that as time goes on a greater and greater proportion of available time would be spent out of the royalty generating activity, that is $a_k^* < a_{k+1}^*$ for all *k*.[7]

The intuition of the above paragraph is fine for a finitely lived artist, although in any case care must be taken. The result does ultimately depend on the parameters of the system. However, with an infinitely long planning horizon, this intuition is not nearly so clear. An infinitely lived agent never becomes "old". However, the infinite horizon model is, generally, accepted as being more relevant since no one really knows the date of his or her death. The use of inter-temporal discounting is often taken as being understood in a stochastic sense – that is, under discounting a unit of utility of next period is valued at *d* today, which can simply interpreted as receiving 1 unit of utility with probability *d*, and in that case infinitely discounted utility is equivalent to expected utility where the probability of survival from one period to the next is *d*.[8] This is a far more realistic interpretation than the option of a finitely lived artist, and so it is the option taken for our model.

To say more about the optimal sequence of time allocations, we need to consider the first order conditions of any two consecutive periods, say periods *k* and $k+1$. From (3) the first order condition for period $k+1$ is just:

$$\begin{aligned} & wu'(c_{k+1}^*) - r\sum_{i=k+1}^{\infty}(dp)^{i-(k+1)}u'(c_i^*) = 0 \qquad k = 1,2,.... \\ & \Rightarrow wu'(c_{k+1}^*) = \left(\frac{r}{dp}\right)\sum_{i=k+1}^{\infty}(dp)^{i-k}u'(c_i^*) \qquad k = 1,2,.... \\ & \Rightarrow dpwu'(c_{k+1}^*) = r\sum_{i=k+1}^{\infty}(dp)^{i-k}u'(c_i^*) \qquad k = 1,2,.... \end{aligned} \tag{4}$$

However, note that (3) can be written as

$$wu'(c_k^*) = ru'(c_k^*) + r\sum_{i=k+1}^{\infty}(dp)^{i-k}u'(c_i^*) \qquad k = 1,2,...$$

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7 This argument does rely upon a "well behaved" utility form, which would generate a monotone sequence of time allocations.

8 See, for example, Mas Collel et al. (1995), pp. 733-34.

Combining this with (4) (and simplifying) we get, for all *k*:

$$\begin{aligned} & wu'(c_k^*) = ru'(c_k^*) + dpwu'(c_{k+1}^*) \\ & \Rightarrow \frac{u'(c_k^*)}{u'(c_{k+1}^*)} = \frac{dpw}{w-r} \end{aligned} \tag{5}$$

From (5), several aspects of the general solution can be noted, as is summarised in the following simple result:

Result 1: The artist chooses an optimal sequence of time allocations such that consumption is increasing (decreasing, constant) over time as dpw is greater than (less than, equal to) $w-r$.

Result 1 is easy to prove, and is due entirely to the assumed concavity of the utility function. *From now on we will only consider the case* $dpw > w-r$, since empirically we are far more likely to observe increasing levels of consumption than the opposite.

However, knowing how the optimal levels of consumption compare to each other sheds little light on how the optimal time allocations compare over time. But, from (1) we have:

$$\begin{aligned} c_k^* &= a_k^* w + r\sum_{i=1}^{k} p^{k-i}(1-a_i^*) \\ c_{k+1}^* &= a_{k+1}^* w + rp\sum_{i=1}^{k+1} p^{k-i}(1-a_i^*) \\ &= a_{k+1}^* w + r(1-a_{k+1}^*) + r\sum_{i=1}^{k} p^{k-i}(1-a_i^*) \end{aligned}$$

Thus, it turns out that

$$\begin{aligned} c_{k+1}^* &= a_{k+1}^* w + r(1-a_i^*) + p(c_k^* - a_k^* w) \\ &= pc_k^* + w(a_{k+1}^* - pa_k^*) + r(1-a_{k+1}^*) \end{aligned}$$

Now, since $r(1-a_{k+1}^*) \geq 0$, we know that

$$\begin{aligned} & c_{k+1}^* \geq pc_k^* + w(a_{k+1}^* - pa_k^*) \\ & \Rightarrow c_{k+1}^* - pc_k^* \geq w(a_{k+1}^* - pa_k^*) \end{aligned}$$

From here, we can state the following:

Result 2: i) $c_{k+1}^* \leq pc_k^* \Rightarrow a_{k+1}^* \leq pa_k^*$

ii) $a_{k+1}^* \geq pa_k^* \Rightarrow c_{k+1}^* \geq pc_k^*$

In words, part i) of result 2 states that if the optimal sequence of consumption is *sufficiently* decreasing over time, then the optimal sequence of time allocation to the non-royalty activity is certainly non-increasing over time (since $pa_k^* \le a_k^*$), and part ii) states that if the optimal sequence of time allocation to the non-royalty activity is *not* sufficiently decreasing over time, then consumption is not very decreasing over time.

We can say more about the optimal sequence of time allocation by reconsidering equation (5). Define

$$\frac{u'(c_k^*)}{u'(c_{k+1}^*)} = \frac{u'\left(a_k^* w + r\sum_{i=1}^{k} p^{k-i}(1-a_i^*)\right)}{u'\left(a_{k+1}^* w + rp\sum_{i=1}^{k+1} p^{k-i}(1-a_i^*)\right)} \equiv h_k(a_k^*, a_{k+1}^*) \quad k = 1, 2, ...$$

Now, we can analyse the effect of an increase in a_k^* on a_{k+1}^*, by simply applying the implicit function theorem to $h_k(a_k^*, a_{k+1}^*)$:

$$\frac{\partial a_{k+1}^*}{\partial a_k^*} = -\frac{\left(\frac{\partial h_k(a_k^*, a_{k+1}^*)}{\partial a_k^*}\right)}{\left(\frac{\partial h_k(a_k^*, a_{k+1}^*)}{\partial a_{k+1}^*}\right)} \quad k = 1, 2, ... \qquad (6)$$

From (1), we can easily calculate

$$\frac{\partial c_k^*}{\partial a_k^*} = \frac{\partial c_{k+1}^*}{\partial a_{k+1}^*} = w - r, \quad \frac{\partial c_k^*}{\partial a_{k+1}^*} = 0, \quad \frac{\partial c_{k+1}^*}{\partial a_k^*} = -rp$$

Therefore;

$$\frac{\partial h_k(a_k^*, a_{k+1}^*)}{\partial a_k^*} = \frac{u''(c_k^*)(w-r)u'(c_{k+1}^*) + u'(c_k^*)u''(c_{k+1}^*)rp}{u'(c_{k+1}^*)^2}$$

$$\frac{\partial h_k(a_k^*, a_{k+1}^*)}{\partial a_{k+1}^*} = \frac{-u'(c_k^*)u''(c_{k+1}^*)(w-r)}{u'(c_{k+1}^*)^2}$$

Dividing the first of these by the second, we reconstruct (6) as

$$\frac{\partial a_{k+1}^*}{\partial a_k^*} = \frac{u''(c_k^*)(w-r)u'(c_{k+1}^*) + u'(c_k^*)u''(c_{k+1}^*)rp}{u'(c_k^*)u''(c_{k+1}^*)(w-r)} \quad k = 1, 2, ...$$

$$= \frac{u''(c_k^*)u'(c_{k+1}^*)}{u'(c_k^*)u''(c_{k+1}^*)} + \frac{rp}{w-r} \quad k = 1, 2, ... \qquad (7)$$

$$= \frac{R_a(c_k^*)}{R_a(c_{k+1}^*)} + \frac{rp}{w-r} \quad k = 1, 2, ...$$

where

$$R_a(c) \equiv -\frac{u''(c)}{u'(c)}$$

is the Arrow-Pratt measure of absolute risk aversion.

From (7), since the utility function is strictly concave by assumption (and so absolute risk aversion is positive), and since by assumption $w > r$, we know that an increase in a_k^* will lead to an increase in a_{k+1}^*. Furthermore, since we are assuming that $wpd > w - r$, so that $c_k^* < c_{k+1}^*$, and if absolute risk aversion is decreasing (as is very often assumed), then $R_a(c_k^*) > R_a(c_{k+1}^*)$, and in that case we would have

$$\frac{\partial a_{k+1}^*}{\partial a_k^*} > 1 \quad k = 1, 2, ...$$

Again, in the interests of empirical reality, *we shall always assume from now on that absolute risk aversion is decreasing*.

Now, let the time allocation in period *k* be defined by the general variable *a*, so that the optimal time allocation in the next period can be found from

$$a_{k+1}^* = s_k(a) \quad k = 1, 2, ...$$

where $s_k(a)$ is defined implicitly by

$$h_k(a, s_k(a)) = \frac{wpd}{w - r} \quad k = 1, 2, ...$$

that is,

$$\frac{u'\left(aw + r(1-a) + r\sum_{i=1}^{k-1} p^{k-i}(1-a_i^*)\right)}{u'\left(s_k(a)w + r(1-s_k(a)) + rp(1-a) + rp\sum_{i=1}^{k-1} p^{k-i}(1-a_i^*)\right)} = \frac{wpd}{w-r} \quad k = 1, 2, ... \tag{8}$$

Now, consider the case of $k = 1$, so that $\sum_{i=1}^{k-1} p^{k-i}(1-a_i^*) = 0$, and so (8) becomes

$$\frac{u'\left(aw + r(1-a)\right)}{u'\left(s_1(a)w + r(1-s_1(a)) + rp(1-a)\right)} = \frac{wpd}{w-r}$$

Since we are assuming that $wpd > w - r$, we get

$$\begin{aligned} &aw + r(1-a) < s_1(a)w + r(1-s_1(a)) + rp(1-a) \\ &\Rightarrow a(w-r) < s_1(a)(w-r) + rp(1-a) \end{aligned} \tag{9}$$

Now, consider the value of *a* for which $s_1(a) = 1$. Substituting into (9), this value of *a* must satisfy $a(w-r) < (w-r) + rp(1-a)$, which reduces to $0 < (1-a)(w - r(1-p))$, which given our assumptions on *w*, *r* and *p*, can only imply

$a < 1$. This all tells us that the graph of $s_1(a)$ certainly goes above the diagonal line in $(a, s_1(a))$ space, that is, there exist values of *a* for which $s_1(a) > a$ (see figure 1).

At the other extreme, consider the value of *a* for which $s_1(a) = 0$. Define this value as $\tilde{a}_1$. Substitute $s_1(a) = 0$ into (9) to get $\tilde{a}(w - r) < rp(1 - \tilde{a})$, that is

$$\tilde{a} < \frac{rp}{w - r(1 - p)}$$

But since the right-hand-side of this is strictly positive $s_1(a)$ can go to 0 at some positive value for *a*, as has been represented in Figure 1.

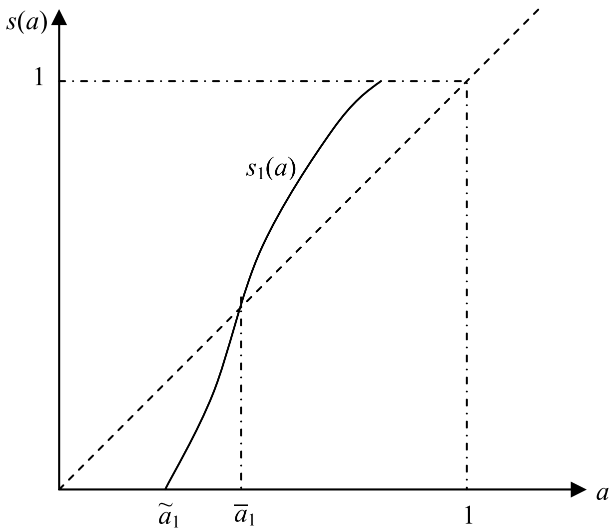

Figure 1

However, we summarise what we know about the function $s_1(a)$ in the following lemma:

Lemma 1: Assuming $dpw > w - r$ and decreasing absolute risk aversion, the function $s_1(a)$ is everywhere increasing with slope greater than 1, and there must exist a value of *a*, say $\bar{a}_1$, such that $s_1(a) > \bar{a}_1$ for all $a > \bar{a}_1$. There also exists a value of *a*, say $\tilde{a}_1$, where $\bar{a}_1 > \tilde{a}_1$, such that for all $a_1^* \le \tilde{a}_1$ we get $a_2^* = 0$.

Finally, consider any two consecutive functions $s_k(a)$ and $s_{k+1}(a)$. Note that if we define that part of royalty earnings that accrues for recording activities before period *k* as $r\sum_{i=1}^{k-1} p^{k-i}(1-a_i^*) \equiv A_k$, then we have

$$\frac{u'\left(aw+r(1-a)+A_k\right)}{u'\left(s_k(a)w+r(1-s_k(a))+rp(1-a)+pA_k\right)}=\frac{wpd}{w-r} \quad k=1,2,...$$

Now, consider the effect of an increase in *k*, holding *a* constant. We get an increase in A_k of $r(1-a)$, and a corresponding effect on $s_k(a)$ that can be calculated by again using the implicit function theorem. Let us simplify our notation with

$$m(s_k, A_k) = aw + r(1-a) + A_k$$
$$n(s_k, A_k = s_k w + r(1-s_k) + rp(1-a) + pA_k$$

Thus, the effect of an increase in *k* can be calculated by

$$\begin{aligned}\frac{\partial s_k}{\partial A_k} &= \frac{u''(m)r(1-a)u'(n)-u'(m)u''(n)pr(1-a)}{u'(m)u''(n)(w-r)} \\ &= \frac{u''(m)r(1-a)u'(n)}{u'(m)u''(n)(w-r)} - \frac{u'(m)u''(n)pr(1-a)}{u'(m)u''(n)(w-r)} \\ &= \frac{R_a(m)r(1-a)}{R_a(n)(w-r)} - \frac{pr(1-a)}{(w-r)} \\ &= \frac{r(1-a)}{(w-r)}\left(\frac{R_a(m)}{R_a(n)} - p\right)\end{aligned}$$

But, again since by assumption $m < n$, and $p < 1$ decreasing absolute risk aversion implies that this is strictly positive. This tells us that, of two consecutive functions $s_k(a)$ and $s_{k+1}(a)$, the function corresponding to period $k+1$ is graphically everywhere above that corresponding to period *k*;

Lemma 2: Assuming $dpw > w - r$ and decreasing absolute risk aversion, as *k* increases, the functions $s_k(a)$ are shifted upwards at all values of *a*.

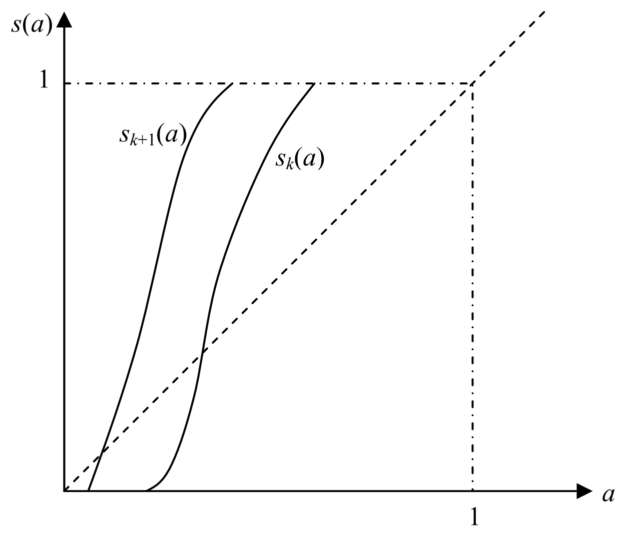

Figure 2

Between lemmas 1 and 2, we have the direct implication that on each function $s_k(a)$ there exists a limit value of *a*, say $\bar{a}_k$, such that $s_k(a) > \bar{a}_k$ for all $a > \bar{a}_k$. They also imply that $\bar{a}_{k+1} < \bar{a}_k$ and $\tilde{a}_{k+1} < \tilde{a}_k$ for all *k*. This is shown in Figure 2.

The importance of the values of $\bar{a}_k$ and $\tilde{a}_k$ for each *k* cannot be overemphasised. For any *k*, if $\tilde{a}_k < a_k^* < \bar{a}_k$ then we get $a_{k+1}^* < a_k^*$, that is a decreasing sequence of time allocations. If $a_k^* < \tilde{a}_k$ then we get $a_{k+1}^* = 0$. On the other hand, if $a_k^* > \bar{a}_k$ then we will have an increasing sequence of time allocations, $a_{k+1}^* > a_k^*$. However, note that since over time the functions $s_k(a)$ shift upwards and to the left, it turns out that the zone of options for a decreasing sequence of time allocations gets gradually squeezed away.

Thus, as time goes on, it becomes more and more likely that the sequence of time allocations is increasing (i.e. more and more time is allocated to the non-royalty generating activity). Of course, once a^* goes to 1, it will stay at 1 for all successive periods. Thus, if the time allocated to non-royalty activities is sufficiently high right at the outset, then we can get an intertemporal equilibrium in which no royalty activity is entered into as of the second period. One reasonable trajectory of time allocations is shown in Figure 3.

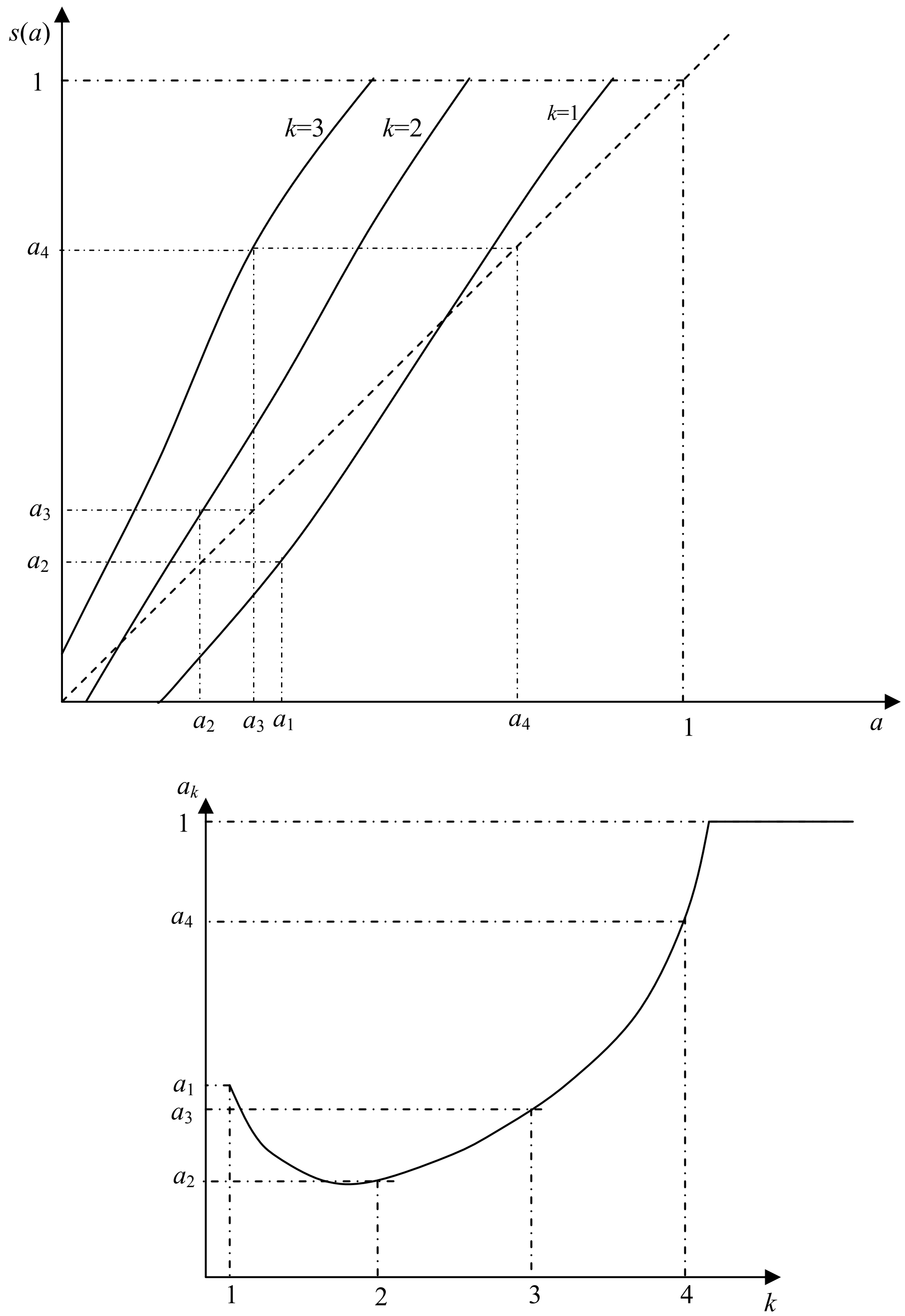

Figure 3

We can imagine a time sequence like that shown in Figure 3 as corresponding to an artist who, while young, dedicates some of her time writing songs for future recording while simultaneously working in other activities for livelihood. Then, she goes to the recording studio thereby reducing her dedication to other activities for a short while. Once a sufficiently large royalty portfolio has been generated, the artist then goes back to spending less time recording and progressively more in other activities (which, as was pointed out earlier in footnote 7, could even include leisure). The type of sequence shown in Figure 3 implies that copyrighted products are created and made available sooner rather than later, something that (assuming that this is in the public interest) seems to imply the existence of a modern day version of the "invisible hand".

4. Comparative statics of the optimal time allocation in period *k*.

It is interesting to consider the comparative statics exercise of exactly how changes in the parameters affect the optimal choice for a_k for any given *k*. Since the choice of a_k^* has intertemporal implications, and so any alteration in a_k^* due to a parameter change always involves both a gain and a loss, and as a result some of the comparative statics effects are not monotone. However, under certain relevant conditions on the utility function, sufficient conditions can be found for the logical effects to hold; an increase in *w* to increase a_k^*, an increase in *r* to decrease a_k^*, and an increase in *d* to decrease a_k^*. The effect of an increase in *p* on a_k^* turns out to be more problematic.

Consider firstly the effect of an increase in *w*. From the implicit function theorem[9]:

$$\frac{\partial a_k^*}{\partial w} = -\frac{\left(\dfrac{\partial^2 J}{\partial a_k \partial w}\right)}{\left(\dfrac{\partial^2 J}{\partial (a_k)^2}\right)} \quad k = 1,2,...$$

But since *J* is concave in a_k the denominator of this is strictly negative for all *k*, we have (from the first order conditions (3)):

$$sign\frac{\partial a_k^*}{\partial w} = sign\frac{\partial^2 J}{\partial a_k \partial w}$$

$$= sign\left[\left(u'(c_k^*) + u''(c_k^*) w \frac{\partial c_k^*}{\partial w}\right) - r\sum_{i=k}^{\infty}(dp)^{i-k} u''(c_i^*)\frac{\partial c_i^*}{\partial w}\right] \quad k = 1,2,...$$

From (1) we have for all *k*

[9] The function *J*(*a*) was defined above at the start of the previous section.

$$\frac{\partial c_k^*}{\partial w} = a_k^*$$

and so it turns out that

$$sign\frac{\partial a_k^*}{\partial w} = sign\left[\left(u'(c_k^*) + u''(c_k^*)wa_k^*\right) - r\sum_{i=k}^{\infty}(dp)^{i-k}u''(c_i^*)a_i^*\right] \quad k = 1,2,...$$

Given the concavity of the utility function, we know for sure that

$$-r\sum_{j=k}^{\infty}(dp)^{i-k}u''(c_i^*)a_i^* > 0$$

and so we can conclude that an increase in *w* will certainly increase each a_k^* if

$$u'(c_k^*) + u''(c_k^*)wa_k^* \geq 0$$

for all *k*. By reordering this, we can conclude that

$$-\frac{wa_k^*u''(c_k^*)}{u'(c_k^*)} \leq 1 \;\Rightarrow\; \frac{\partial a_k^*}{\partial w} > 0 \qquad k = 1,2,...$$

Clearly, this condition is satisfied the less risk averse is our artist. But in particular, (1) indicates that $c_k > wa_k$, and so

$$-\frac{wa_k^*u''(c_k^*)}{u'(c_k^*)} < -\frac{c_k^*u''(c_k^*)}{u'(c_k^*)}$$

The right-hand-side of this is just the measure of relative risk aversion, and so we can conclude the following:

Result 3: Relative risk aversion uniformly less than or equal to 1 is a sufficient condition for an increase in *w* to increase the amount of time dedicated to non-royalty generating activities in all periods.

Secondly, consider the effect of an increase in *r*. Logic points to an increase in the royalty wage decreasing the amount of time in each period dedicated to the other activity (live performing). However, again we shall see that we need to condition utility to get the result. Following the same sorts of steps as above, we get:

$$sign\frac{\partial a_k^*}{\partial r} = sign\left[u''(c_k^*)w\frac{\partial c_k^*}{\partial r} - \sum_{i=k}^{\infty}(dp)^{i-k}\left(u'(c_i^*) + ru''(c_i^*)\frac{\partial c_i^*}{\partial r}\right)\right] \quad k = 1,2,...$$

However, from (1) we find that for all *k*,

$$\frac{\partial c_k^*}{\partial r} = \sum_{i=1}^{k}p^{k-i}(1-a_i) > 0$$

Therefore, from the concavity of the utility function it holds that

$$u''(c_k^*)w\frac{\partial c_k^*}{\partial r} < 0$$

and so it is sufficient to get the desired result that, for all i greater than or equal to k

$$u'(c_i^*) + ru''(c_i^*)\frac{\partial c_i^*}{\partial r} \geq 0$$

$$\Rightarrow \; u'(c_i^*) + u''(c_i^*) r \sum_{j=1}^{i} p^{i-j}(1-a_j) \geq 0$$

However, again this condition reorders to something more useful:

$$-\frac{r\sum_{j=1}^{i} p^{i-j}(1-a_j) u''(c_i^*)}{u'(c_i^*)} \leq 1 \quad i = k, k+1, \ldots \;\Rightarrow\; \frac{\partial a_k^*}{\partial r} < 0 \quad k = 1, 2, \ldots$$

And since from (1) $c_i > r\sum_{j=1}^{i} p^{i-j}(1-a_j)$, we have the same condition as previously:

Result 4: Relative risk aversion uniformly less than or equal to 1 is a sufficient condition for an increase in r to decrease the amount of time dedicated to live performing in all periods.

Next, consider the effect of an increase in the intertemporal discount factor, d. We have:

$$sign\frac{\partial a_k^*}{\partial d} = sign\left[u''(c_k^*)w\frac{\partial c_k^*}{\partial d} - rp\sum_{i=k}^{\infty}(i-k)(dp)^{i-k-1}u'(c_i^*) - r\sum_{i=k}^{\infty}(dp)^{i-k}u''(c_i^*)\frac{\partial c_i^*}{\partial d}\right]$$

But from (1) we have (for all k)

$$\frac{\partial c_k^*}{\partial d} = 0$$

Thus, our condition is simply

$$sign\frac{\partial a_k^*}{\partial d} = sign\left[-rp\sum_{i=k}^{\infty}(i-k)(dp)^{i-k-1}u'(c_i^*)\right]$$

However, clearly this sign is unambiguously negative (since $i > k$);

Result 5: An increase in d has the unambiguous effect of decreasing a_k^* (i.e. increasing the time spent recording at the expense of time spent in live performing).

Result 5 is rather logical – the more patient is the artist, the more she will want to record, since that pays off as a future income stream, and this income stream gives greater utility the higher is the intertemporal discount factor. Surprisingly, it turns out although the same intuition works well for the case of an increase in *p*, it cannot be proven formally without further conditioning upon the problem. We would expect that the lower is the rate of royalty payment depreciation ($1-p$), the more attractive it is to spend time in the royalty generating activity. Thus logic tells us that an increase in *p* should decrease a_k^* for all *k*. Let's see.

The implicit function theorem and the first order condition tell us that:

$$sign\frac{\partial a_k^*}{\partial p} = sign\left[u''(c_k^*)w\frac{\partial c_k^*}{\partial p} - r\sum_{i=k}^{\infty} p^{i-k-1}d^{i-k}\left((i-k)u'(c_i^*) + pu''(c_i^*)\frac{\partial c_i^*}{\partial p}\right)\right]$$

Now, from (1) we have for all *k*:

$$\frac{\partial c_k^*}{\partial p} = r\sum_{i=1}^{k}(k-i)p^{k-i-1}(1-a_i) > 0$$

Thus, from the concavity of the utility function, it turns out that

$$u''(c_k^*)w\frac{\partial c_k^*}{\partial p} < 0$$

Therefore, in order for the increase in p to decrease a_k^*, it is sufficient that

$$-r\sum_{i=k}^{\infty} p^{i-k-1}d^{i-k}\left((i-k)u'(c_i^*) + pu''(c_i^*)\frac{\partial c_i^*}{\partial p}\right) \le 0$$

Clearly, this term will indeed be non-positive if the term in brackets is non-negative for all *i* greater than *k*:[10]

$$(i-k)u'(c_i^*) + pu''(c_i^*)\frac{\partial c_i^*}{\partial p} \ge 0 \quad i > k$$

Substituting for the value of the effect of p on period *i* consumption, we have a sufficient condition of

$$(i-k)u'(c_i^*) + u''(c_i^*)r\sum_{j=1}^{i}(i-j)p^{i-j}(1-a_j) \ge 0 \quad i > k$$

which we write as

[10] When $i = k$, this term reduces to $-ru''(c_k^*)\partial c_k^*/\partial p$, which when coupled with the first term of the entire equation ($wu''(c_k^*)\partial c_k^*/\partial p$) gives us $(w-r)u''(c_k^*)\partial c_k^*/\partial p$ which is strictly negative since we are assuming that $w > r$.

$$1 \geq -\frac{r\sum_{j=1}^{i}(i-j)p^{i-j}(1-a_j)u''(c_i^*)}{(i-k)u'(c_i^*)} \quad i > k$$

Again, this is akin to a condition that limits relative risk aversion to being something less than 1, but this time in order that it be sufficient for relative risk aversion to be everywhere not greater than 1, we would require that

$$c_i^* = wa_i^* + r\sum_{j=1}^{i} p^{i-j}(1-a_j) \geq \left(\frac{1}{i-k}\right) r\sum_{j=1}^{i}(i-j)p^{i-j}(1-a_j) \quad i > k$$

Note that, since $wa_k^* \geq 0$, it is clearly sufficient that:

$$\sum_{j=1}^{i} p^{i-j}(1-a_j) \geq \left(\frac{1}{i-k}\right)\sum_{j=1}^{i}(i-j)p^{i-j}(1-a_j) \quad i > k$$

$$\Rightarrow \sum_{j=1}^{i} p^{i-j}(1-a_j) \geq \sum_{j=1}^{i}\left(\frac{i-j}{i-k}\right)p^{i-j}(1-a_j) \quad i > k$$

$$\Rightarrow \sum_{j=1}^{i}\left(\frac{i-j}{i-k}-1\right)p^{i-j}(1-a_j) \leq 0 \quad i > k$$

$$\Rightarrow \sum_{j=1}^{i}\left(\frac{k-j}{i-k}\right)p^{i-j}(1-a_j) \leq 0 \quad i > k$$

$$\Rightarrow \frac{1}{i-k}\sum_{j=1}^{i}(k-j)p^{i-j}(1-a_j) \leq 0 \quad i > k$$

And since $i > k$, again we can reduce our sufficient condition to

$$\sum_{j=1}^{i}(k-j)p^{i-j}(1-a_j) \leq 0 \quad i > k \tag{10}$$

Result 6: If (10) is satisfied, and if relative risk aversion is less than or equal to 1, then an increase in the royalty discount rate will decrease a_k^* for all *k* (i.e. it will lead to a greater proportion of time dedicated to recording.

5. Discussion

In this section we offer some intuition regarding our results, and we discuss the assumptions used in sections 3 and 4. Firstly, it may seem paradoxical that the condition $dpw > w - r$ implies that, over time, the artist will dedicate less and less of her time to the royalty generating activity (recording) and more and more to earning the spot wage (live performing). This may seem paradoxical since the condition that generates the result is easier to satisfy the greater is *r* and the greater is *p* – so the greater is the base royalty wage, or the more the royalty is retained over periods, the less the artist wants of it as time goes on. However, the apparent paradox is easily explained, and it can in fact be seen that such behaviour is indeed reasonable.

Firstly, note that the absolute size of the royalty wage rate (and the other parameters) is what ties down the way time is split between the two activities in any given period, not how this split changes over time. Secondly, when the royalty wage is high, it seems reasonable that the artist would like to take advantage of this over her lifetime as much as possible. But that is exactly what happens when over time a growing share of time is dedicated to the non-royalty generating activity, since this implies that it is in the initial periods that more time is spent recording, setting up a royalty portfolio that then generates income for more future periods. A strategy that involves an artist dedicating relatively more of her time to recording is a strategy that generates more copyrights. If the existence (and distribution) of copyrights is also socially desirable, then a strategy in which, as periods go by, the allocation of time is spent in the non-royalty generating activity is growing, is also socially desirable – such a situation implies more copyrights are generated earlier.

It is valid to ask how robust the final conclusions are to more realistic assumptions. Firstly, we could allow the artist to save money from one period to another (either positive savings, or negative savings – that is, loans) at a constant interest rate. This would complicate the model, since the choice variable would now be two-dimensional in each period. It would, however, allow the artist to have a greater overall utility position in the optimum.

Secondly, we could allow for uncertainty as to the royalty income stream. This would certainly alter the model significantly, and it would become very complex. The model would also lose time consistency, that is, as uncertainty unravels, the optimal vector of time allocations would change. In any case, to some extent artists are insured against royalty uncertainty, since they typically receive an advance on the royalties due from a work, which appears as a spot payment on account. This appears to be an important feature of artists' contracts and may offer better incentives to each party to commit (see Towse, 2001). It also clearly mitigates the effects of royalty uncertainty upon the time allocation decision. However, if royalty uncertainty were to be properly introduced, two simple intuitions would likely arise – more risk averse artists would spend relatively less time in the royalty generating lottery, and decreasing absolute risk aversion would contribute to a decreasing sequence of optimal time dedication (that is, as time goes on, more and more time dedicated to the royalty activity), since as royalty income mounts up, a decreasingly risk averse individual will naturally take on more risk.[11]

Thirdly, we could also include reputation effects on the two per-unit-time wage rates. That is, the more time that is spent in the royalty generating activity (gaining a reputation) it is reasonable to expect that the per-unit-time wage rates would increase. If both were to increase in the same proportion, nothing at all would change in the model, but if they were to increase in different proportions, the optimal trajectory would certainly be altered.

Fourthly, we could make utility a two-dimensional function, with leisure being the second element. That is, in the above model, we have implicitly held the proportion of leisure in the non-royalty generating activity constant. However, if leisure were included in a more direct manner, then it would be interesting to consider how the total amount of time that would be dedicated to work would evolve over time. This would be a very interesting element to include, but would again be very complex, as the decision again would become two-dimensional in each period. However, we can state one simple intuition – if leisure were a normal good, then the more time is spent in the royalty generating activity, the more time would be spent into the future on leisure (and less time working), since it is the royalty generating activity that causes future income to rise.

Fifthly, artists' skills are differentiated and most do not become superstars. It is therefore worth asking whether the 'bread–and–butter' artist would make the same decision as the star.

Finally, it remains to draw some conclusions for copyright policy-makers: what are the implications of this simple model for variables that law reform can control, such as the scope and duration of copyright. Perhaps the most important conclusions are that if the optimal intertemporal sequence of consumptions is decreasing, then we at least have a good chance that the intertemporal sequence of time allocations is increasing, i.e. the artist spends relatively more time when young on creating copyrights. This has the socially beneficial function of allowing more copyrights to be enjoyed earlier and for longer. Thus the government may want to control the only

[11] The question of risk-aversion on the part of artists has been raised in the paper; it certainly seems from real world observations that artists prefer royalty contracts to spot payments even through the

variable that it can to achieve such an outcome – the fraction of the royalty income that is not lost to depreciation, *p*. Thus the model provides some support for strong copyright protection.[12] Also, since we have shown that increasing *r*, decreasing *w*, or increasing *d* will also all have the effect of increasing the amount of time in all periods dedicated to creating copyrights (recall, some of these results are conditional upon the form of utility, or risk aversion, assumed), then again assuming that this is socially beneficial, the government could investigate bringing about such changes, perhaps by regulating the amount of each tax on the different sources of income.[13]

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former appear to be less favourable to them (see, for example, Liebowitz (1987) for an account of this type of behaviour). Artists' supply decisions are certainly complex ones!

[12] The support is not, of course, outright, since one would have to take into account the social costs of increased protection over time as well (see, for example, Landes and Posner (1989)).

[13] Exactly how *d* can be regulated is not at all clear.