Software piracy and social welfare: an analysis of protection mechanisms and pricing strategies

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Abstract
Based on a set-up where two competing firms produce software products with respective support packs, we analyze a government's choice of optimal level of monitoring and penalty for social welfare maximization. After simulating the pricing strategies we observe that social welfare and consumers' surplus are maximized at competitive, and profits are maximized at monopoly price levels. By using the monitoring rate the government can balance the distribution of welfare among users and producers of software. When monitoring rates are at deterrent levels, increasing the penalty level results in increased firm profits, whereas total consumers' surplus and social welfare decrease.

JEL classification: D40; K42; L10; L86

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1. Introduction

Intellectual property has been facing the risk of being reproduced without the authorization of its owner since the beginning of industrialization. Having realized this fact, developed countries enacted the Paris (1883) and Berne (1886) Conventions, which both contained clauses concerning intellectual property protection. Today, the TRIPS Agreement states that "The protection and enforcement of intellectual property rights should contribute to the promotion of technological innovation and to the transfer and dissemination of technology, to the mutual advantage of producers and users of technological knowledge and in a manner conducive to social and economic welfare, and to a balance of rights and obligations" (TRIPS Agreement, Article 7).

Piracy, which is one of the most obvious and harmful examples of intellectual property rights violation, is a phenomenon that affects both firms and users significantly. Especially in the software market volume of piracy is at drastic levels, since it is very easy to make copies with minimal cost. Government's role in the software market is to promote on one hand the development of advanced products by providing firms the environment to experience profits, while at the same time to improve consumer's welfare (economic and social welfare in the TRIPS statement above for producers and users of software, respectively).

The 2002 worldwide piracy rate is estimated to be 39% (in the US it was 23%) amounting to retail worldwide revenue loss of $13 billion. Alone in the United States the total loss of revenue in the same year was almost $2 billion, which resulted in a loss of more than 105,000 jobs, amounting to $5.3 billion in lost wages and more than $1.4 billion in tax revenue lost (BSA).

Our analysis investigates the social welfare effects of intellectual property protection mechanisms introduced to eliminate software piracy. We concentrate on mechanisms
designed to protect intellectual property developed by software producers, where the
government controls the two mechanisms, namely, monitoring and penalizing
infringers. This paper's contribution to the existing literature on intellectual property
rights protection and software piracy is twofold. First, we analyze the government's
decision of social welfare maximization and the distribution of social welfare among
firms and users by considering the selection of optimal level of monitoring and
penalty. The parameter space for monitoring rates is divided into four regions, which
helps us to analyze effects of currently exercised monitoring rates and also social
welfare maximizing monitoring rates. Second, we simulate the pricing strategies of
two competing software producers when they do and do not offer similar products to
the market, and with simulations we are able to separately analyze the effects of
software and support packs of differing quality.

Our paper is similar in terms of its content, assumptions and set-up to the recent
papers of Shy and Thisse (1999), Yoon (2002), Banerjee (2003), and Chen and Png
pirate where network externalities are present, and concludes that the market structure
will be determined by the government's selection of monitoring and penalty levels.

Chen and Png (2003) state that although reducing the price and increasing the
monitoring rate are substitute strategies to stimulate demand for the original software,
these strategies have different effects on social welfare. The authors conclude that
social welfare is improved by reducing prices rather than increasing the monitoring
rate. Yoon (2002) seeks the optimal level of copy protection in his model. By using a
model similar to Novos and Waldman (1984), contrary to the general claim, Yoon
argues that an increase in copyright protection may decrease or increase the social
welfare loss due to underutilization. Shy and Thisse (1999) analyze software piracy
under network externalities, and policy instruments are not included in their model. If
the network externality is weak, then firms protect their software and get higher
profits. On the other hand if the network externality is strong enough, prices and profit
levels are higher when firms do not protect their software.
Unlike Chen and Png (2003) and Banerjee (2003), in our model, we do not consider
network effects. We let the government set the monitoring rate and penalty whereas in
Chen and Png firms determine the monitoring rate. The aim of the monitoring policy
is different in our model and the one of Chen and Png, where we use monitoring to
balance the welfare of producers and consumers, while Chen and Png use it to
maximize firm profits. Our model allows discretion in the sense that the government
may choose among a set of deterrent levels of monitoring and penalty that enables the
redistribution of welfare among agents in the economy without changing the total
welfare. Also, differing from the single firm model of Chen and Png, our model
consists of two price-setting and profit maximizing firms in the presence of a
competitive market structure. We also differ from Shy and Thisse (1999) in the sense
that whereas in their and our model the software industry consists of two price-setting
firms selling differentiated software packages, Shy and Thisse investigate the effects
of software protection policies under weak and strong network externalities. Their
model and our model classify users as being either support-oriented or support-
independent, but monitoring and penalty are not included in Shy and Thisse.
Recently, Yoon (2003) specializes in analyzing underutilization and underproduction
whereas we do not.
This paper is organized as follows. In Section 2 we present the set-up of our model.
Following Section 3, where we solve the model, in Section 4 we present our
simulations. We conclude in Section 5 and Section 6 gives suggestions on how the
model presented in this paper can be extended. The proofs of Lemma 1 and 2 are shown in Appendix A.

2. The model

We develop a model of a software industry under presence of a government. The industry consists of two profit-maximizing firms producing two differentiated software packages. We denote these software packages by A and B with the respective prices $p_A$ and $p_B$. Once the products are developed, production of additional software packages is assumed to be costless for both firms, since the technology enables firms to make identical copies easily and efficiently. However, due to availability of CD and DVD writers and "Peer to Peer" software like Kazaa or i-Mesh, pirated copies are also identical to originals. The only difference between the pirated software package and the original one is the support pack bundled in the original software. The support pack may contain printed manuals, how-to books and technical support via phone or on the Internet for buyers of the original software package.

Besides Microsoft Office and Corel Office, or econometrics software packages, another example to our model is the digital encyclopedia market. Britannica and Encarta are similar products regarding content and prices, but do provide different support packs like frequency of on-line updates, Encarta's local editions, or Encarta's provision of access to MSN channels and Britannica's provision of access to magazines like The Economist and Newsweek (Alevizou, 2002).

We assume that the registration code (that comes with software purchase) which activates the support pack can only be used for a single installation. None of the benefits of a support pack can be enjoyed by the pirating parties. We denote the extra
utility provided by the support pack by $s_A$ for software package A, and by $s_B$ for software package B.

The government's role in the model is regulation and enforcement. Government sets the level of monitoring (which mostly takes the form of ex-parte searches or criminal raids to identify infringers) $\mu$, and the penalty level $f$ that is charged to pirates when piracy is detected. While setting the level of monitoring, the government aims to maximize social welfare. The government incurs the costs associated with monitoring $C(\mu)$, and collects the penalty $f$ from infringers caught. This penalty level is a fixed amount independent from the quality of a software.

The software packages are doing the same task, but are not perfect substitutes. These products are differentiated in the sense that each software package has some superiority over the other. The advantages of using one of the software over the other may be associated with the design, functionality and user friendliness of the software packages. All of these advantages add up to the quality of the software package. The quality of the software package A is denoted by $A$ and is uniquely determined by Firm A. Similarly, the quality of software package B is denoted by $B$ and is uniquely determined by Firm B.

The firms are located at each end of the unit interval, Firm A is located at zero, and Firm B at 1. The value attached to software package A by consumers is maximized at zero and is equal to $A$, with $A > 0$. Similarly, the value attached to software package B is maximized at 1 and is equal to $B$, with $B > 0$. Given the quality levels, each firm sets its profit-maximizing price. We assume that the marginal cost of production is zero.

In the first stage of a three-stage game, the government announces the monitoring rate and the penalty level that will be charged to pirates caught. In the second stage, after
observing government's policy and determination against infringers (μ and f are
exogenous to firms), firms determine and announce the qualities of their software
packages and support packs, and set their profit-maximizing prices simultaneously.
And in the last stage, users decide on either to enter the game by buying or pirating a
single unit of software, or not to enter the game and leave with their reservation utility
of zero.

In the model we have two types of consumers. The first type, the support dependent
consumers, denoted by x, gain extra utility from the support pack provided by
software manufacturers. In that sense they are strict buyers, and they either buy
software or do not. They are uniformly distributed along the unit interval [0,1]. The
second type of consumers, the support independent consumers, denoted by y, do not
utilize the support pack. These advanced users do not need the support pack at all and
this makes them potential pirates. They may either pirate, buy software, or not buy,
and are also uniformly distributed along the unit interval [0,1]. In sum the number of
users is equal to 2.

Both types of consumers vary in their valuation of the software packages. Consumers
attaching higher value to software package A are positioned closer to zero. Likewise,
consumers attaching higher value to software package B are positioned closer to 1.
Consumers have to choose among five options: they can buy software A, buy
software B, pirate software A, pirate software B, or do without any software package.
Both types of consumers are risk-neutral.
2.1. Utility profiles of support dependent users

Support dependent users are strict buyers. They either buy one unit of a software package or do without it. Let \((1 - x)A\) be the value attached to software package A by the consumer at position \(x\) in the unit interval. If a support dependent consumer located at \(x\) buys software package A at price \(p_A\), he will get his valuation of \((1 - x)A\) plus a constant utility \(s_A\) from utilizing the support pack yielding the net utility of \((1 - x)(A + s_A) - p_A\). Similarly, net utility of a user who buys software B is given by \(x(B + s_B) - p_B\).

We can determine the location of a marginal user on the unit interval by setting up the individual rationality and incentive compatibility constraints. The marginal consumer \(x_{AW}\) who is indifferent between buying A and doing without is derived from

\[(1 - x)(A + s_A) - p_A \geq 0 \quad \text{(at strict equality)}\]

as

\[x_{AW} = \frac{A + s_A - p_A}{A + s_A}. \quad (1)\]

Similarly, the marginal consumer \(x_{AB}\) who is indifferent between buying A and B is derived from \((1 - x)(A + s_A) - p_A \geq x(B + s_B) - p_B\) as

\[x_{AB} = \frac{A - p_A + p_B + s_A}{A + s_A + B + s_B}. \quad (2)\]

The marginal consumer \(x_{BW}\) who is indifferent between buying B and doing without is obtained from \(x(B + s_B) - p_B \geq 0\) as

\[x_{BW} = \frac{p_B}{B + s_B}. \quad (3)\]

2.2. Utility profiles of support independent users

Net utility of a support independent user who prefers to buy software A is given by

\((1 - y)A - p_A\). On the other hand, the expected utility received by a support
independent user who pirates software A is given by the difference of utility received from piracy minus penalty paid if the user is caught as \((1 - \mu)(1 - y)A - \mu f\). Similarly, the utility of a user who buys software B is given by \(yB - p_B\), while the expected utility of a user who pirates B is equal to \((1 - \mu)yB - \mu f\).

We can compute the location of a marginal user \(y_{\text{AW}}\) who is indifferent between buying A and doing without from \((1 - y)A - p_A \geq 0\) as

\[
y_{\text{AW}} = \frac{A - p_A}{A}. \tag{4}\]

By using the constraint \((1 - \mu)(1 - y)A - \mu f \geq 0\), we can derive the location of the marginal user \(y_{\text{AW}}\) who is indifferent between pirating A and doing without as

\[
y_{\text{AW}}^\wedge = 1 - \frac{\mu f}{(1 - \mu)A}. \tag{5}\]

We derive the location of the marginal user \(y_{\text{AB}}\) who is indifferent between buying A and buying B using \((1 - y)A - p_A \geq yB - p_B\) as

\[
y_{\text{AB}} = \frac{A + p_B - p_A}{A + B}. \tag{6}\]

The location of the marginal user \(y_{\text{AA}}\) indifferent between buying A and pirating A is derived from the constraint \((1 - y)A - p_A \geq (1 - \mu)(1 - y)A - \mu f\) as

\[
y_{\text{AA}} = 1 - \frac{p_A - \mu f}{\mu A}. \tag{7}\]

We can derive the location of the marginal user \(y_{\text{AB}}\) indifferent between buying A and pirating B from \((1 - y)A - p_A \geq (1 - \mu)yB - \mu f\) as

\[
y_{\text{AB}} = \frac{A - p_A + \mu f}{A + (1 - \mu)B}. \tag{8}\]
while the marginal user \( y_{AB} \) indifferent between pirating A and buying B is derived from \((1 - \mu)(1 - y)A - \mu f \geq yB - p_B\) as

\[
y_{AB} = \frac{(1 - \mu)A + p_B - \mu f}{(1 - \mu)A + B}.
\]  

Similarly, using \( yB - p_B \geq 0 \) we can derive the marginal user \( y_{BW} \) indifferent between buying B and doing without as

\[
y_{BW} = \frac{p_B}{B},
\]  

and the marginal user \( y_{BB} \) indifferent between buying B and pirating B from

\[
y_{BB} = \frac{p_B - \mu f}{\mu B}.
\]  

We can derive the location of the marginal user \( y_{BW} \) who is indifferent between pirating B and doing without from \((1 - \mu)yB - \mu f \geq 0\) as

\[
y_{BW} = \frac{\mu f}{(1 - \mu)B}.
\]  

And lastly, the marginal user \( y_{AB} \) indifferent between pirating A and pirating B is given from \((1 - \mu)(1 - y)A - \mu f \geq (1 - \mu)yB - \mu f\) by

\[
y_{AB} = \frac{A}{A + B}.
\]  

3. Solving the model

We summarize the linear (expected) utility function for each option as:
\[
\begin{align*}
(1-x)(A + s_A) - p_A & \quad \text{if a support dependent consumer buys A} \\
x(B + s_B) - p_B & \quad \text{if a support dependent consumer buys B} \\
(1-y)A - p_A & \quad \text{if a support independent consumer buys A} \\
(1-\mu)(1-y)A - \mu f & \quad \text{if a support independent consumer pirates A} \\
yB - p_B & \quad \text{if a support independent consumer buys B} \\
(1-\mu)yB - \mu f & \quad \text{if a support independent consumer pirates B} \\
0 & \quad \text{if a consumer does without any software}
\end{align*}
\]  

(14)

3.1. Demand profile of support dependent users

When Firm A and Firm B charge competitive prices, demand profiles of both firms cover most of the support dependent consumers. Since in the unit interval all consumers located to the left of \( x_{AB} \) buy software package A, the demand Firm A faces is \( x_{AB} \). Similarly, all consumers located to the right of \( x_{AB} \) buy software package B, and the corresponding demand for software B is \( 1 - x_{AB} \).

The respective demand functions for both software are given by

\[
D_{SD}^A = \begin{cases} 
  x_{AB} & \text{if } 0 < x_{BW} \leq x_{AW} \\ 
  x_{AW} & \text{if } 0 < x_{AW} < x_{BW} \\ 
  0 & \text{if } x_{AW} \leq 0 
\end{cases}, \quad \text{and} \quad \\
D_{SD}^B = \begin{cases} 
  1 - x_{AB} & \text{if } x_{BW} \leq x_{AW} < 1 \\ 
  1 - x_{BW} & \text{if } x_{AW} < x_{BW} < 1 \\ 
  0 & \text{if } 1 \leq x_{BW} 
\end{cases}.
\]

When \( x_{BW} < x_{AW} \), using (2) the demand functions for both software are given by

\[
D_{SD}^A = \frac{A - p_A + p_B + s_A}{A + s_A + B + s_B},
\]  

(15)

and

\[
D_{SD}^B = \frac{B - p_B + p_A + s_B}{A + s_A + B + s_B}.
\]  

(16)
These demand functions are symmetric in the sense that they are increasing in the value of the software package A and B, respectively, and decreasing in the price difference \((p_A - p_B)\) and \((p_B - p_A)\), respectively, between the software packages.

On the contrary, when \(x_{AW} < x_{BW}\) so that demand profiles of Firm A and Firm B do not clash, using (1) and (3) the demand functions are defined as

\[
D_{SD}^A = \frac{A + s_A - p_A}{A + s_A},
\]

(17)

and

\[
D_{SD}^B = \frac{B + s_B - p_B}{B + s_B}.
\]

(18)

The last two demand equations reveal that when \(x_{AW} < x_{BW}\), each demand function is independent of the other software package’s value and price. When prices \(p_A\) and \(p_B\) are very high, this case occurs, and firms’ demand profiles are restricted only to consumers that assign high values to software packages.

### 3.2. Welfare of support dependent consumers

Support dependent consumers buying software package A lie in \([0, x_{AB}]\). We sum the expected net benefit of these consumers over \([0, x_{AB}]\) to get the total surplus where we define \(p_A^E\) and \(p_B^E\) as the equilibrium prices of software packages A and B, respectively. Hence, the total surplus of support dependent consumers buying software package A is given by the integral

\[
\int_0^{x_{SD}^E} [(1-x)(A + s_A) - p_A^E]dx.
\]

(19)
Depending on the price charged by the producer, either (15) or (17) will be used as $D_{A}^{SD, E}$ in (19).

Similarly, total surplus of support dependent consumers buying software package B is given by the integral
\[ D_{B}^{SD, E} \int_{0}^{\infty} [x(B + s_{B}) - p_{B}^{E}]dx. \]  

Depending on the price levels, either (16) or (18) will be used as $D_{B}^{SD, E}$ in (20).

3.3. Demand and welfare analysis of support independent consumers

Support independent consumers have five alternatives to choose from: buying A, buying B, pirating A, pirating B, and doing without. We have defined $y_{AW}$ in (4) as the marginal consumer indifferent between buying A and doing without. Up to $y_{AW}$, consumers buying software package A gain non-negative utility. This implies that the support independent consumers buying A must lie in $[0, y_{AW}]$. The same reasoning leads us to the fact that support independent consumers pirating software package A must lie in $[0, y_{AW}]$. Similarly, support independent consumers buying software package B lie in $[y_{BW}, 1]$, whereas consumers pirating software package B lie in $[y_{BW}, 1]$. These four points $y_{AW}$, $y_{BW}$, $y_{AW}$ and $y_{BW}$ must also lie along the unit interval. Remaining marginal users defined in the previous sections lie between these four points. In the following computations we do if any of these four points is calculated to be less than zero, then it is set equal to zero. On the other hand, if any of
these four points exceeds 1, then it is set equal to 1. Making these simplifications keep these points in the unit interval without loss of generality.

Each ordering of $y_{AW}$, $y_{BW}$, $y_{AW}^\wedge$ and $y_{BW}^\wedge$ gives us different demand functions.

These four points can be ordered in 24 different ways.

**Lemma 1.** If $y_{AW} < y_{AW}^\wedge$, then a support independent user will never pirate software package $A$. If $y_{BW} < y_{BW}^\wedge$, then a support independent user will never pirate software package $B$.

We exclude the orderings stated in Lemma 1 and rearrangement results in 14 different orderings.

The expressions for marginal users defined above are functions of values and prices of software packages and support packs, the level of monitoring and the penalty level. Having $y_{AW}$ close to 1 means that Firm A charges a low price relative to the value of the software package $A$. Thus, the greater $y_{AW}$, the larger the demand profile for software package $A$. Likewise, the smaller $y_{BW}$, the larger the demand profile for software package $B$. If the monitoring level and penalty are deterrent, demand for pirated software packages will decrease, and $y_{AW}^\wedge$ will be closer to zero and $y_{BW}^\wedge$ will be closer to 1.

Now, we will sketch the demand functions associated with each of these orderings.

We define $D_A^U$ and $D_B^U$ as support independent users’ demand for original software packages $A$ and $B$, respectively. $R_A$ and $R_B$ are defined to be the support independent
users’ demand for pirated software packages A and B, respectively. In the derivation of the demand functions we use the price levels.

Lemma 2. We say that Firm A charges a high price if \( p_A > \mu[(1-y)A + f] \). When Firm A charges a high price, no support independent consumer buys software package A. Similarly, we say that Firm B charges a high price if \( p_B > \mu(yB + f) \) and no support independent consumer buys software package B at that price.

When \( p_A \leq \mu[(1-y)A + f] \) and \( p_B \leq \mu(yB + f) \), we say that Firm A and Firm B charge low prices. So as to extend the demand profiles of software packages, Firm A and Firm B may charge low prices.

3.4. Total demand estimation corresponding to each case

We obtain total demand for any type of software by adding the support dependent and the support independent users' demand functions. Since for each case stated above (regarding the support independent users) we will be facing different values for location of buyers, pirates and users who opt to stay out, we need to calculate separate total demand functions by analyzing each case.

As an example we provide Case A and one of its subcases (Case A.3.). Solutions for remaining cases may be obtained from the authors upon request.

In Case A we have the ordering of marginal types as \( y_{BW} < y_{BW} < y_{AW} \). If Firm B charges a low price defined as \( p_B \leq \mu(yB + f) \), for the subcase \( y_{BB} \leq y_{AB} \) (which implies that \( y_{BW} \leq y_{BB} \leq y_{BW} \)) we can calculate the expected utility of each user by
making use of Figure 1. (Analyses done with use of figures are valid for all parameter values used in Section 5.)

Fig. 1. Expected utility curves for case A.3.

In Figure 1 we observe that at every point up to $y_{AW}$, buying A brings strictly more expected benefit than pirating A. Therefore, in this subcase no support independent consumer will pirate software package A. In order to find the demand functions, we trace the utility curves from zero to 1 and pick up the utility curve that brings more net expected benefit than the others. Starting from zero up to $y_{AB}$, buying A brings the highest benefit, since its utility curve lies above all of the remaining utility curves. From $y_{AB}$ to 1, buying B brings the highest benefit since its utility curve lies above all of the other utility curves. Thus, in this subcase support independent users buy either software package A or software package B, and they never pirate any software. This
implies that there is no pirating and we have \( R_A = R_B = 0 \). From (6) support independent users’ demands for original software packages are given as

\[
D_{AB}^{SI} = y_{AB} = \frac{A + p_B - p_A}{A + B},
\]

\[
D_{AB}^{SI} = 1 - y_{AB} = \frac{B - p_B + p_A}{A + B}.
\]

After adding up (15) and (21), we obtain the total demand for original software package A as

\[
D_A = \frac{A - p_A + p_B + s_A}{A + s_A + B + s_B} + \frac{A + p_B - p_A}{A + B} \]

\[
= 2A^2 + 2AB + 2(p_B - p_A)(A + B) + s_A(2A + B + p_B - p_A) + s_B(A - p_A + p_B). \]

Respectively, from (16) and (22) we obtain the total demand for original software package B as

\[
D_B = \frac{B - p_B + p_A + s_B}{A + s_A + B + s_B} + \frac{B - p_B + p_A}{A + B} \]

\[
= 2B^2 + 2AB + 2(p_A - p_B)(A + B) + s_B(2B + A + p_A - p_B) + s_A(B - p_B + p_A). \]

Firm A and Firm B maximize their profit functions \( \Pi_A \) and \( \Pi_B \) with respect to \( p_A \) and \( p_B \), respectively. Although we assume the marginal cost of production to be zero, there are also sunk costs associated with the value of the software and the support pack. In our analysis we assume that sunk costs do not constitute a major factor in profit calculation, and they are excluded from the profit functions. Firm profits for all cases are defined as

\[
\Pi_A = p_A D_A, \quad \text{(25)}
\]

and

\[
\Pi_B = p_B D_B. \quad \text{(26)}
\]
Now, we can get the profit functions by substituting (23) and (24) into (25) and (26).

We define consumers' surplus as the difference between the value attached to a software package and the price charged for the software package. For users who pirate software, consumers' surplus is defined to be the expected benefit of pirating. We define $CS_A$ as the total surplus of consumers buying and pirating software package A and calculate it as

$$CS_A = \int_0^{R_A} [(1-x)(A+s_A) - p_A^E]dx + \int_0^{R_A} [(1-y)A - p_A^E]dy + \int_0^{R_A} [(1-\mu)(1-y)A - \mu f]dy.$$  

(27)

Since we have $R_A = 0$ for this case, we end up with

$$CS_A = \int_0^{D_A^{0,\infty}} [(1-x)(A+s_A) - p_A^E]dx + \int_0^{D_A^{\infty,\infty}} [(1-y)A - p_A^E]dy.$$  

(28)

Similarly, $CS_B$ is the total consumers' surplus of consumers buying or pirating software package B.

$$CS_B = \int_0^{D_B^{0,\infty}} [x(B+s_B) - p_B^E]dx + \int_0^{D_B^{\infty,\infty}} (yB - p_B^E)dy + \int_0^{R_B} [(1-\mu)yB - \mu f]dy.$$  

(29)

Since $R_B = 0$, we end up with

$$CS_B = \int_0^{D_B^{0,\infty}} [x(B+s_B) - p_B^E]dx + \int_0^{D_B^{\infty,\infty}} (yB - p_B^E)dy.$$  

(30)

We define social welfare, $SW$, as the sum of total consumers' surplus and firm profits minus the monitoring cost $C(\mu)$.

$$SW = CS_A + CS_B + \Pi_A + \Pi_B - C(\mu).$$  

(31)
4. Model simulation

It is not possible to calculate analytically the optimal price levels for the 14 cases, since taking derivatives of profit functions with respect to prices deliver complex first order condition systems. Therefore, we run simulations to estimate these values by computer. The source code of the simulation is written in Pascal, and the simulation output is compiled in Microsoft Excel.

First, we set the values of software packages \((A, B)\), the values of the support packs \((s_A, s_B)\), and the penalty level \(f\). \(A\) and \(B\) take values within the set of \(\{1.0, 1.1\}\), whereas the support packs \(s_A\) and \(s_B\) take values within the set of \(\{0.1, 0.2\}\). Next, we define the vector of monitoring rates between \([0.15; 0.40]\) increasing by \(0.025\). The reason for choosing the high starting value of 0.15 for monitoring level is explained in Section 4.1. The software price levels \(p_A\) and \(p_B\) both take values within the domain of \([0.1; 0.7]\). After generating the matrix of price pair combinations by increasing prices by \(0.0025\), we then merge them with the monitoring rates to set-up the \((p_A, p_B, \mu)\) combinations. Next, we define all the marginal users derived in the previous sections and calculate their values using the \((A, B, s_A, s_B, p_A, p_B, \mu, f)\) combinations. If for a price-support pack-penalty combination the computed value of a marginal user is greater than 1 or less than zero, we set it equal to 1 or zero, respectively. The subcases can be defined using the theoretical results derived earlier.

Using the parameter combinations we calculate the marginal users, and by looking at their ordering we find the respective case that fits to that combination. Once we find the case we next look for the subcase for that particular combination. When we find the subcase, we can derive the demand functions (calculated analytically) belonging to that subcase. We run these simulations in Pascal for a total of 638,891 cases \([\approx 241\text{ cases}]\).
Then, for each case we calculate the profits, consumers' surplus and social welfare based on the demand functions, and transfer our results to Microsoft Excel.

In the model the government first sets the monitoring level $\mu$ and based on this exogenous monitoring rate firms select their optimum prices. Since there can be several price alternatives (and hence different profit levels) coming from different (sub)cases belonging to the same monitoring rate, firms play their best responses to maximize their profits. By iteration we find the Nash prices of each firm, then find the optimal levels of $(p_A^E, p_B^E, \mu, f)$, and finally calculate the optimal profit, consumers' surplus and social welfare levels. In the simulations we take cost of monitoring as $C(\mu) = \frac{\mu^3}{3}$ (see Chen and Png (2003)), and observe that it constitutes a negligible proportion of total social welfare. Therefore, for simplicity we set $C(\mu) = 0$. At the end of this process we obtain the tables used in the following analyses.

4.1. Simulation results for Case 1 (the "Base Case")

<Values used: $A = B = 1.0, s_A = s_B = 0.1, f = 1.0$>

When $0.15 \leq \mu < 0.15$ and $0.375 \leq \mu$, we observe that firms charge monopoly prices due to either very low or very high monitoring rates. Hence, assigning additional values to $\mu$ beyond $0.15 < \mu < 0.375$ does not contribute to our discussion. From the simulation results we see that the parameter space can be divided into four regions defined as $\mu \leq 0.15, 0.15 < \mu < 0.35, 0.35 \leq \mu < 0.375$ and $\mu \geq 0.375$. We name these four regions defined over $\mu$ as Region 1, Region 2, Region 3 and Region 4, respectively.
Proposition 1. Given \( A = B = 1.0, s_d = s_g = 0.1, f = 1.0 \), total consumers' surplus and social welfare are maximized in Region 2 where firms charge competitive prices. In Region 4 firms charge monopoly prices and highest firm profits are obtained within this region. Prices of both firms are identical at all monitoring levels.

The numerical proof of Proposition 1 is given in Table 1.

Table 1

Simulation Results for \( A = B = 1.0, s_d = s_g = 0.1, f = 1.0 \)

| \( \mu \) | \( p_A \) | \( p_B \) | \( D_A \) | \( D_B \) | \( R_A \) | \( R_B \) | \( \Pi_A \) | \( \Pi_B \) | Total \( \Pi \) | CS | SW |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.150 | 0.5500 | 0.5500 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.2750 | 0.2750 | 0.5500 | 0.7875 | 1.3375 |
| 0.175 | 0.2625 | 0.2625 | 1.0000 | 1.0000 | 0.0000 | 0.0000 | 0.2625 | 0.2625 | 0.5250 | 1.0750 | 1.6000 |
| 0.200 | 0.3000 | 0.3000 | 1.0000 | 1.0000 | 0.0000 | 0.0000 | 0.3000 | 0.3000 | 0.6000 | 1.0000 | 1.6000 |
| 0.225 | 0.3375 | 0.3375 | 1.0000 | 1.0000 | 0.0000 | 0.0000 | 0.3375 | 0.3375 | 0.6750 | 0.9250 | 1.6000 |
| 0.250 | 0.3750 | 0.3750 | 1.0000 | 1.0000 | 0.0000 | 0.0000 | 0.3750 | 0.3750 | 0.7500 | 0.8500 | 1.6000 |
| 0.275 | 0.4125 | 0.4125 | 1.0000 | 1.0000 | 0.0000 | 0.0000 | 0.4125 | 0.4125 | 0.8250 | 0.7750 | 1.6000 |
| 0.300 | 0.4500 | 0.4500 | 1.0000 | 1.0000 | 0.0000 | 0.0000 | 0.4500 | 0.4500 | 0.9000 | 0.7000 | 1.6000 |
| 0.325 | 0.4875 | 0.4875 | 1.0000 | 1.0000 | 0.0000 | 0.0000 | 0.4875 | 0.4875 | 0.9750 | 0.6250 | 1.6000 |
| 0.350 | 0.5375 | 0.5375 | 0.9625 | 0.9625 | 0.0000 | 0.0000 | 0.5375 | 0.5375 | 1.0347 | 0.5264 | 1.5611 |
| 0.375 | 0.5500 | 0.5500 | 0.9500 | 0.9500 | 0.0000 | 0.0000 | 0.5500 | 0.5500 | 1.0450 | 0.5025 | 1.5475 |
| 0.400 | 0.5500 | 0.5500 | 0.9500 | 0.9500 | 0.0000 | 0.0000 | 0.5500 | 0.5500 | 1.0450 | 0.5025 | 1.5475 |

In Region 1 both firms charge monopoly prices of \( p_d = (A + s_d)/2 \) and \( p_g = (B + s_g)/2 \), and all support dependent users buy and all support independent users pirate software. Increasing the monitoring rate in this region will not affect firm profits because prices are constant and an increase in monitoring rate within this low monitoring range is not sufficient to make pirating support independent users buy any software package. In real life we notice that especially in developing countries monitoring rates are very low whereas piracy rates are very high. In Region 1 we observe that all of the support independent users (50% of total users) pirate when
\( \mu \leq 0.15 \). Whereas there are countries experiencing extreme rates of piracy like Vietnam (95%), China (92%), Russia (89%), Indonesia (89%), there are also countries like Canada (39%) and Italy (47%) with lower piracy rates (IIPA a). Even though we cannot easily differentiate between support dependent and independent users' ratio in a country, still, the relatively low piracy rates of the above mentioned countries may represent the users' ratio of our model.

The reason for starting the simulations at \( \mu = 0.15 \) is because even at this high rate of monitoring all of the support independent users pirate software, and hence for levels below this rate we will observe the same profit and consumers' surplus levels. In our model to be able to derive the social welfare maximizing levels of monitoring and penalty, we define the upper boundary of Region 1 at the monitoring level of \( \mu = 0.15 \). This region is a representation of currently exercised monitoring rates where the number of raids conducted in China were 3 (in 2002), in Russia 81 (in 2002), and in Italy 223 (in 2002) (IIPA b).

In Region 2, firms charge competitive prices of \( p_A = p_B = \mu f + \mu AB/(A + B) \) and increasing the monitoring rate will increase software prices, which can be shown as \( \partial p_A / \partial \mu = \partial p_B / \partial \mu = f + AB/(A + B) > 0 \). In this region the equilibrium prices are set such that every support independent user buys either software package A or B, and there is no piracy. Since demand is constant and prices are increasing in monitoring rate, profits also increase in monitoring rate. However, as prices increase, consumers' surplus decreases. In this region, decreases in total consumers' surplus are matched with increases in firm profits. Thus, by changing the monitoring rate \( \mu \), the government can reshape the distribution of welfare between firms and consumers.

However, although firm profits plus total consumers' surplus is constant throughout the region, social welfare will decrease when the government increases monitoring.
rate due to the cost component (which is ignored in our model). Depending on the characteristics of a country, the government can shape the distribution of welfare. If it is a developing country with low levels of computer usage, the government can increase the welfare of consumers to promote use of software packages. In the latter stages of development, the government can support firms by reshaping the distribution of welfare in favor of them to foster technological development and promote the creativity of new and improved software.

In Region 3, following the increase in prices some buyers decide not to buy any software package and since their expected utility from piracy is negative at these high monitoring rates, they also do not pirate. Firm profits still increase compared with the levels in Region 2, but total consumers’ surplus decreases for the reason explained above. This also results in a decrease in social welfare compared with that of Region 2.

In Region 4, where the government sets high levels of monitoring $\mu \geq 0.375$, prices increase up to monopoly price levels with no piracy occurring. Here, consumers choose among buying any software package or doing without any software. Since the prices increase up to monopoly prices and stay constant thereafter, demand, profit levels and consumers' surplus also behave the same way.

The question that arises here is which level of monitoring should the government choose? Every $\mu$ chosen in Region 2 results in higher values of social welfare than that of Region 1, Region 3 and Region 4. Hence, if the government is looking for maximum social welfare, in equilibrium it should set $0.175 \leq \mu < 0.375$. But which value of $\mu$ should the government choose? Again, it depends on the characteristics of the country. Going back to the TRIPS Agreement statement quoted in Section 1, social welfare mentioned in the statement corresponds to total consumers' surplus in
our model, whereas *economic welfare* corresponds to firm profits. Thus, a government needs to balance the benefits of producers and users according to the needs of the country. The ratio of total consumers’ surplus (which also includes utility gained from piracy) to total firm profits may be a target variable in decision-making. If this ratio is above a pre-specified value, then the incentives of firms to develop new software will lessen. On the contrary, if the ratio is less than the specified value, consumers will enjoy low level of benefits. The government also needs to keep in mind that producers will be discouraged in developing new products if it sets monitoring rates at levels that correspond to low profits.

Now, we investigate the consequences of changes in model parameters.

4.2. Value of $A$ increases

<Values used: $A=1.1$, $B=1.0$, $s_A=s_B=0.1$, $f=1.0$>

Suppose that Firm A produces a software that features advanced tools and hence has better quality than the other software.

**Proposition 2.** Compared with the base case, when one of the firms produces a higher quality software package, the profit level of the firm producing higher quality software increases in all regions, whereas the other firms’ profit level decreases in Regions 2, 3 and 4, and stays the same in region 1. Total consumers’ surplus, total profits and social welfare increase in all regions. Whereas in Regions 2 and 3 both firms charge the same prices (except for $\mu = 0.250$), in Regions 1 and 4 Firm A charges higher prices.

The proof of Proposition 2 is given in Table 2.
By looking at the simulation results we observe that following the higher value of software produced by Firm A, in Regions 2 and 3, Firm A either charges a price close to \( p_B \) or a slightly larger price to capture additional users even though Firm A produces a software with higher quality. Compared with the levels of the base case, this strategy results in increased (decreased) demand and higher (lower) profits in Region 2 and Region 3 for Firm A (Firm B). In Region 4 Firm A charges prices higher than those of the base case but still captures additional buyers which results in extra profits. Total profits, consumers' surplus and social welfare all increase in all regions compared with the base case.

Although Firm B's strategy is charging prices close to those of Firm A as best responses, in Regions 2, 3 and 4 demand for software B and profits of Firm B decrease compared with the base case. In this case Firm A acts as the price setter and Firm B as the follower, and the end results is increased demand for software A. Firm A follows a "wise-pricing" strategy where increased price is followed by an increase in demand.

| \( \mu \) | \( p_A \) | \( p_B \) | \( D_A \) | \( D_B \) | \( R_A \) | \( R_B \) | \( \Pi_A \) | \( \Pi_B \) | Total \( \Pi \) | CS | SW |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.150 | 0.6050 | 0.5500 | 0.5000 | 0.5000 | 0.5238 | 0.4762 | 0.3025 | 0.2750 | 0.5775 | 0.8349 | 1.4124 |
| 0.175 | 0.2675 | 0.2675 | 1.0433 | 0.9476 | 0.0043 | 0.0048 | 0.2791 | 0.2535 | 0.5326 | 1.1464 | 1.6790 |
| 0.200 | 0.3050 | 0.3050 | 1.0465 | 0.9512 | 0.0011 | 0.0012 | 0.3192 | 0.2901 | 0.6093 | 1.0714 | 1.6807 |
| 0.225 | 0.3425 | 0.3425 | 1.0476 | 0.9524 | 0.0000 | 0.0000 | 0.3588 | 0.3262 | 0.6850 | 0.9964 | 1.6814 |
| 0.250 | 0.3825 | 0.3800 | 1.0409 | 0.9540 | 0.0051 | 0.0000 | 0.3981 | 0.3625 | 0.7607 | 0.9188 | 1.6795 |
| 0.275 | 0.4200 | 0.4200 | 1.0445 | 0.9489 | 0.0031 | 0.0035 | 0.4387 | 0.3985 | 0.8372 | 0.8414 | 1.6787 |
| 0.300 | 0.4575 | 0.4575 | 1.0465 | 0.9512 | 0.0011 | 0.0012 | 0.4788 | 0.4352 | 0.9140 | 0.7664 | 1.6804 |
| 0.325 | 0.4950 | 0.4950 | 1.0476 | 0.9524 | 0.0000 | 0.0000 | 0.5386 | 0.4714 | 0.9900 | 0.6914 | 1.6814 |
| 0.350 | 0.5375 | 0.5375 | 1.0352 | 0.9387 | 0.0000 | 0.0000 | 0.5564 | 0.5045 | 1.0610 | 0.6066 | 1.6676 |
| 0.375 | 0.5750 | 0.5500 | 0.9903 | 0.9370 | 0.0000 | 0.0000 | 0.5694 | 0.5154 | 1.0848 | 0.5568 | 1.6416 |
| 0.400 | 0.5750 | 0.5250 | 0.9794 | 0.9728 | 0.0000 | 0.0000 | 0.5632 | 0.5107 | 1.0739 | 0.5806 | 1.6545 |
4.3. Increasing both $A$ and $B$

<Values used: $A=B=1.1$, $s_A=s_B=0.1$, $f=1.0$>

Now, suppose that both firms decide to improve the quality of software they produce by equal amounts.

**Proposition 3.** Following identical increases in the quality of both software products, compared with the base case, both firms' profits, consumers' surplus and social welfare increase in all regions. Both firms charge higher prices in all regions, and at some monitoring levels demands decrease.

The proof of Proposition 3 is given in Table 3.

Table 3

Simulation Results for $A=B=1.1$, $s_A=0.1$, $s_B=0.1$, $f=1.0$

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$p_A$</th>
<th>$p_B$</th>
<th>$D_A$</th>
<th>$D_B$</th>
<th>$R_A$</th>
<th>$R_B$</th>
<th>$\Pi_A$</th>
<th>$\Pi_B$</th>
<th>Total $\Pi$</th>
<th>CS</th>
<th>SW</th>
</tr>
</thead>
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<td>0.6050</td>
<td>0.6050</td>
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<td>0.5000</td>
<td>0.5000</td>
<td>0.5000</td>
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<td>0.3025</td>
<td>0.6050</td>
<td>0.3025</td>
<td>0.6050</td>
</tr>
<tr>
<td>0.175</td>
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<td>0.2725</td>
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<td>0.9935</td>
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<td>0.0065</td>
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</table>

We observe that when both firms offer products with higher quality, prices increase but this results in positive and negative changes in demand at different monitoring levels. However, the net effect is increased profits for both firms in all regions where
consumers also benefit from improved products, and social welfare increases in all regions. Both firms' pricing strategy here is increasing prices. At $\mu = 0.400$ firms charge prices lower than monopoly levels, since at that monitoring rate the case parameter values fit in results in prices of either 0.575 or 0.635. The equilibrium occurs at 0.575.

In the first three simulations it was remarkable that at all monitoring levels except for $\mu = 0.150$ the percentages of pirates were really small. We can explain this by stating that in our set-up the support dependent users do buy software in any case, whereas in reality people pirate even though they need assistance with use of software and cannot use software effectively. Since the total number of users in our model is equal to 2, at a maximum half of them can pirate software. And 50% of support independent users pirate in the base case. At the high monitoring rates piracy is almost prevented due to negative expected return for potential pirating users. We also investigate monitoring levels much higher than 0.150 to find out the social welfare maximizing monitoring rate.

**4.4. Increasing $s_d$**

<Values used: A=B=1.0, $s_A=0.2$, $s_B=0.1$, f=1.0>

Imagine that one of the firms decides on increasing the value of support pack it offers to buyers. The consequences can be summed up in the following proposition.

**Proposition 4.** Following an increase in the value of the support pack of Firm A offers, compared with the base case, in Regions 2 and 3 profits of Firm A (Firm B) increase (decrease) while total profits remain unchanged, and consumers' surplus and social welfare increase. In Regions 1 and 4 profits of Firm B stay the same but
Firm A's profits and hence total profits, consumers' surplus and social welfare all increase. Firms charge the base case prices in Regions 2 and 3, and Firm A charges higher prices in Regions 1 and 4.

The proof of Proposition 4 is given in Table 4.

Table 4
Simulation Results for $A=B=1.0$, $s_A=0.2$, $s_B=0.1$, $f=1.0$

<table>
<thead>
<tr>
<th>$\mu$</th>
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<th>$p_B$</th>
<th>$D_A$</th>
<th>$D_B$</th>
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<th>$R_B$</th>
<th>$\Pi_A$</th>
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<th>Total $\Pi$</th>
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<td>0.5225</td>
<td>1.0625</td>
<td>0.5063</td>
<td>1.5688</td>
</tr>
</tbody>
</table>

In Region 1, following the increase in $s_A$, total consumers' surplus and social welfare increase. In Region 2, Firm A's strategy is charging the same price as Firm B which is the base case price. Now, the prices of software packages are the same but Firm B provides less support to its consumers. Thus, some support dependent consumers switch to buying software package A instead of software package B. Since in Region 2 all support dependent consumers buy either software package A or software package B, an increase in value of support pack $s_A$ causes a profit transfer from Firm B to Firm A, causing the total profit to stay at the level of the base case. At the high levels of monitoring of Region 4, firms again charge monopoly prices, and Firm A reflects the increase in $s_A$ with an increase in the price of the software it produces. This decreases the demand for software A, but still the profit effect coming from the
increase in price dominates the profit loss from the decrease in demand, and Firm A’s profit, total profit, consumers’ surplus and social welfare all increase. Compared with the results of the base case, we can conclude that a single-side increase in the support pack will result in a transfer of profit from one firm to another without changing the level of total profit in Region 2, and will increase the total consumers’ surplus and consequently the social welfare in all regions.

4.5. Increases in both $s_A$ and $s_B$

<Values used: $A=B=1.0$, $s_A=s_B=0.2$, $f=1.0$>

Now, suppose that both firms increase values of the support packs they provide by the same rate.

Proposition 5. Following increases in the values of support packs both firms offer, compared with the base case, total profits in Regions 1 and 4, and consumers’ surplus and social welfare in all regions increase. Firms charge identical prices in all regions (and the base case prices in Regions 2 and 3).

The proof of Proposition 5 is given in Table 5.

Table 5

Simulation Results for $A=B=1.0$, $s_A=s_B=0.2$, $f=1.0$

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$p_A$</th>
<th>$p_B$</th>
<th>$D_A$</th>
<th>$D_B$</th>
<th>$R_A$</th>
<th>$R_B$</th>
<th>$\Pi_A$</th>
<th>$\Pi_B$</th>
<th>Total $\Pi$</th>
<th>CS</th>
<th>SW</th>
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<td>1.0800</td>
<td>0.5100</td>
<td>1.5900</td>
</tr>
</tbody>
</table>
Following increases in the values of both support packs, in Regions 1 and 4 both firms charge monopoly prices, and this results in increased total profits, but also in increases in consumers' surplus and social welfare. In Regions 2 and 3, where firms still charge the competitive prices of the base case, we do not observe any changes from the base case except for an increase in consumers' surplus and social welfare which is due to increased value of support packs to support dependent consumers. As in the base case, social welfare is maximized in Region 2, and firm profits are maximized in Region 4. Compared with the case when only $s_A$ increases, as expected, the parallel increase in $s_B$ is reflected positively as increases on Firm B's profits, total profits, consumers' surplus and social welfare in Regions 1 and 4. In Regions 2 and 3 total consumers' surplus and social welfare increase.

From Tables 1 to 5 it is remarkable that (based on our set-up) due to high value placed on software, increases in value of software result in piracy. On the contrary, increases in value of support pack do not cause piracy.

Now, we also have the tools to compare the consequences of increases in the value of the software and the support pack. We observe that the effects are different. An increase in the value of software A results in increased total profit in all regions, whereas when there is an increase in the support pack Firm A provides, we do not observe increased profits in Regions 2 and 3. Similarly, increases in the values of both software are followed by increased profits in all regions, but when values of both support packs increase, total profits only increase in Regions 1 and 4.

**Proposition 6. If the government targets improving social welfare through subsidizing software or support pack development, compared with the base case,**
whereas quality improvements in both increase consumers' surplus and social welfare in all regions, higher valued software results in higher profits in all regions. On the contrary, higher valued support packs result in higher profits only in Regions 1 and 4.

Please refer to Tables 1 to 5 for the proof of Proposition 6.

Keeping in mind that highest social welfare levels are obtained in Region 2, we can state that if the government targets improving social welfare through subsidizing the software and/or the support pack, motivation of firms for improved products is achieved by supporting improvements in the software packages firms develop. Certainly, we have not incorporated cost of developing software and support pack, but we can consider subsidies as monetary transfers net of costs. This would imply that a government subsidy should target increasing the value of the software rather than the support pack, since the first directly motivates firms by higher profit levels in all regions whereas the latter does not.

4.6. Increase in $f$

<Values used: $A=B=1.0$, $s_a = s_u = 0.1$, $f=1.5 >$

As stated before, the government's choice variables are monitoring and penalty level. Now, we would like to analyze the consequences of an increase in the penalty level.

**Proposition 7.** For deterrent levels of monitoring, when we increase the penalty level keeping everything else constant at the levels of the base case, firm profits increase in all regions whereas total consumers' surplus and social welfare decrease at several monitoring rates. At all monitoring levels firms charge the same prices.
The proof of Proposition 7 is given in Table 6.

Table 6

Simulation Results for $A = B = 1.0$, $s_A = s_B = 0.1$, $f = 1.5$

<table>
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<th>$p_B$</th>
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<th>$D_B$</th>
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<td>0.5225</td>
<td>1.0450</td>
<td>0.5025</td>
<td>1.5475</td>
</tr>
</tbody>
</table>

When the penalty level is increased from 1.0 to 1.5, compared with the other cases, the striking change observed in the equilibrium outcomes is that firms charge monopoly prices at even lower levels of monitoring ($\mu \geq 0.275$). Unlike the cases when $f = 1$, we now have two regions. At $\mu < 0.275$ (Region A) firms charge competitive prices and at $\mu \geq 0.275$ (Region B) firms charge monopoly prices.

Characteristics of Region 1 of the previous cases are not observed here, because the penalty level is too high, and even at the very low levels of monitoring, high penalty levels force support independent consumers to buy original software packages. At the end, high penalty levels induce competition between Firm A and Firm B, and firms do not charge monopoly prices at low levels of monitoring. It is also remarkable that when $\mu \geq 0.275$ we obtain all the results of the base case in Region 4.

When we compare the price levels with those of the base case with $f = 1$, we observe
that for each level of monitoring price levels increase as penalty level increases (except for $\mu = 0.150$). Higher penalty level enables firms to charge higher prices in each region, and with constant demand, this results in higher profits for firms. On the other hand, higher prices lower the total surplus of both support independent and support dependent consumers (except for $\mu = 0.150$). This decrease in the level of total consumers' surplus is matched by the increase in the profit levels. Hence, when we increase the penalty level from $f = 1$ to $f = 1.5$, the level of social welfare in Region A is at the level of Region 2, and the level of social welfare in Region B is at the level of Region 4 of the base case.

For every monitoring rate chosen in Region A, social welfare level realized is higher than that of Region B. Hence, the government could decrease the monitoring rate and cut down monitoring costs (ignored in our model) when it introduces high penalty levels. But this comes at the price of lower consumers' surplus. In sum, by adjusting the penalty rate, the government has another tool besides the monitoring rate to balance the welfare among users and firms with piracy totally eliminated.

A comparison of social welfare levels of the presented cases with the base case is shown in Figure 2.

Fig. 2. Social welfare levels of different cases.
5. Conclusion

In a setting with two firms that produce differentiated software packages with respective support packs we have analyzed the effects of changing the monitoring rate, penalty level, values of software and support packs on market equilibrium and social welfare. Once the penalty level and the level of monitoring are predetermined, firms simultaneously set the values of their software and the respective support packs, observe the rival's selections and then play their best responses by announcing their prices. Our general finding is that when cost of monitoring is ignored, the government that sets the monitoring rate and penalty level can adjust the distribution of welfare among users and producers of software. This provides a tool to the central authority to foster research and development while also considering consumers' surplus.

Based on the simulations ran, in the "base case" consumers' surplus and social welfare are maximized at competitive prices whereas maximum profits occur at monopoly prices. Improvements in the quality of software and support pack may have different effects on firm profits, consumers' surplus and social welfare. A government can prefer to subsidize the software rather than the support pack, since the first results in higher profits at competitive levels, where the latter does not. And when monitoring rates are set at deterrent levels, increasing the penalty level while keeping everything else constant results in increased firm profits and elimination of piracy, but total consumers' surplus in almost all and social welfare at some monitoring levels decrease.
6. Extensions of the model

The paper can be extended to include network effects by modifying the set-up such that the model can still be analytically solved, and if not, then simulated. Also, monitoring cost can be incorporated to the model and simulations and a more detailed analysis can be made where government will incur costs with increased monitoring levels. Costs of producing advanced products by software firms can also be added in the model. All of these modifications demand additional analytical and computational complexity and were beyond the reach of this paper at the time it was written.

Acknowledgements

We thank Ismail Saglam and Fikret Adaman for invaluable suggestions.

Appendix A

Proof of Lemma 1

Suppose that \( y_{AW} < y_{AW} \). Then, from (4) and (5)

\[
1 - \frac{\mu f}{(1 - \mu)A} < \frac{A - p_A}{A}.
\]  

(A.1)

This is equal to

\[
\frac{\mu f}{1 - \mu} > p_A.
\]  

(A.2)

For some support independent users to pirate software A we need to have,

\[
(1 - \mu)(1 - y)A - \mu f > (1 - y)A - p_A.
\]  

And once this equation is solved we get

\[
p_A > \mu[A(1 - y) + f].
\]  

(A.3)

When we combine (A.2) and (A.3) we obtain \( \frac{\mu f}{1 - \mu} > p_A > \mu[A(1 - y) + f] \). And this implies
\[
\frac{\mu f}{1-\mu} > \mu[A(1-y) + f]. \tag{A.4}
\]

This inequality contradicts our assumption about \((1-\mu)(1-y)A - \mu f \geq 0\). Hence, no support independent user pirates software A if \(y_{AW} < y_{AW}\). Similarly, if \(y_{BW} < y_{BW}\), then using (10) and (12) it can be shown that no support independent consumer will pirate software B.

**Proof of Lemma 2**

For some support independent users to buy software A rather than pirate it, we need to have: \((1-y)A - p_A \geq (1-\mu)(1-y)A - \mu f\), which implies that

\[
p_A \leq \mu[(1-y)A + f]. \tag{A.5}
\]

Hence, when \(p_A > \mu[(1-y)A + f]\), no support independent consumer buys software A. Similarly, no support independent user buys software B if

\[
p_B > \mu(yB + f). \tag{A.6}
\]

**References**


International Intellectual Property Alliance (IIPA) a. "SPECIAL 301" decisions on intellectual property. Available on-line at:


Trade-Related Aspects of Intellectual Property Rights (TRIPS) Agreement.
