Copyright protection and innovation in the presence of commercial piracy*

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Abstract

This paper uses a strategic entry-deterrence approach to address the effects of anti-commercial piracy policies on a firm's incentive to innovate. Monitoring increases the firm's incentive to innovate. However, inclusion of innovation does not necessarily result in monitoring as the socially optimal policy. If monitoring is the socially optimal policy then the commercial pirate's entry may or may not be deterred. The entry-deterring limit price and quality is less than that in the monopoly case. Only in the extreme situation the monopoly results are restored.

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1. INTRODUCTION

Commercial piracy has emerged as one of the leading global challenges faced by software businesses, entertainment industry, law enforcement agencies, and international trade partners.[1] This issue assumes importance not only because of the high magnitude of the immediate loss in retail sale but also of its possible detrimental effects on the incentive to innovate.[2] As a consequence, the government is often called upon to perform the duty of strengthening and enforcing copyright. The US Trade Representative in its recent 2004 Special 301 Report, a trade sanction tailored for intellectual property trade concerns, posited that "ineffective enforcement of intellectual property rights, commercial piracy - in particular the growing problem of pirate production of optical media such as CDs, DVDs and CD-ROMs,…continue to be a global threat." The threat is so real as to force the Bush Administration to launch an inter-agency initiative called the *Strategy Targeting Organized Piracy* (STOP) in October 2004.

An accurate assessment of the efficacy of copyright protection must take into consideration the benefits from developing new products as well as the costs associated with administering copyright protection. The general literature on commercial and end-user piracy predominantly focuses on pricing strategies, anti-piracy policies and their impact on piracy, but avoids the issue of innovation and treats it as a sunk cost.[3] In this paper we study the intertwined strategic interaction

[1] Commercial piracy refers to a situation where a firm/s illegally reproduces and sells copies of legitimate products thereby competing with the original producer.
[2] The Business Software Alliance (BSA) in their 2005 Piracy Study claims US$34 billion in worldwide losses. BSA further projects that in the next five years almost US$200 billion worth of software will be pirated globally. In terms of the reduced incentive to innovate, BSA believes that "local software industries crippled from competition with high-quality pirated software" and International Federation of the Phonographic Industry (IFPI) in its 2005 Commercial Piracy Reports argues, "The illegal music trade is destroying creativity and innovation, eliminating jobs and livelihoods and bankrolling organised crime."
[3] See Banerjee (2003, 2006a, 2006b) for the existing literature on commercial piracy. Alternate to commercial piracy is end-user piracy where copying is for personal consumption rather than for

among the copyright enforcing government, the innovating and price-setting monopolist, a potential entrant (hereafter, referred to as the pirate) who practices commercial piracy, and a set of taste-heterogeneous consumers. The purpose of our paper is two-fold. First, we analyze the effects of increased copyright protection on the social welfare loss due to underproduction in the shadow of commercial piracy. In other words, we study the impact of this form of piracy on the incentive to innovate or to create. Second, we investigate whether the socially optimal copyright protection policy results in piracy or in its deterrence as the subgame perfect equilibrium in a full-scale strategic entry-deterrence framework.[4]

In our model the government chooses its copyright protection policy in the form of monitoring, which is costly, and punishing the commercial pirate. The monopolist observes the government's policy and chooses a price and the quality of its product. The monopolist incurs a cost to develop the product. His price and quality strategy either allows (hereafter, referred to as the *accommodating strategy*) or deters (hereafter, referred to as the *aggressive strategy*) the pirate's entry. The pirate after observing the government's policy and the monopolist's strategy makes its entry

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commercial purposes. Chen and Png (1999) show that pricing rather than monitoring is a better strategy for a firm in dealing with piracy by end-users. Cheng, Sims, and Teegen (1997) and Noyelle (1990) mention that the high price of software products is the dominant reason for piracy. Harbough and Khemka (2000) compare targeted enforcement to extensive enforcement and show that the latter is better than the former. Shy and Thisse (1999) show that in the presence of network externalities non-protection against piracy is an equilibrium. Takeyama (1994), Conner and Rumelt (1991), and Nascimento and Vanhonacker (1988) also discuss the role of network externalities on the marketing of software. In the context of digital music, Duchene and Waelbroeck (2007) focus on the welfare effects of increased copyright protection on alternative distribution technologies but do not address the issue of incentive to innovate. Stronger copyright protection increases the profits of firms practicing the conventional sales and marketing of physical products (dubbed as "information-push" technology). On the contrary, stronger copyright protection lowers the profits of firms who adopt the information-pull technology by which consumers search and sample music through the p2p network and decides whether to purchase a quality-enhanced original. In this case consumers respond to a more hostile downloading environment by tuning down searching and sampling effort which in turn reduces sales of the originals and social welfare.

[4] This model encompasses the basic elements emphasized by Landes and Posner (1989: page 326). Furthermore, the social welfare maximizing equilibrium strategy is fully characterized in our model while previous papers have mostly performed comparative statics.

decision. The social welfare maximizing copyright protection policy endogenously determines whether piracy exists in equilibrium or not.

Novos and Waldman (1984) and Bae and Choi (2006) address the issue of incentive to innovate in the presence of end-user piracy. Novos and Waldman (1984) only considers the price-quality combination that allows copying and show that a sufficient condition is needed to sustain the common claim that increases in copyright protection decreases the social welfare loss due to underproduction. [5] Bae and Choi (2006) discern the effects of increased copyright protection into two distinctive costs: a constant across-the-board reproduction cost and a degradation cost which is proportional to consumers' heterogeneous valuation. They consider copy-deterring limit pricing and copying regimes and show that the presence of piracy lowers product quality. In the copying regime increased copyright protection in the form of higher degradation cost unambiguously reduces social welfare.[6]

Qiu (2006) considers the impact of legal and copyright protection policies on software development. He shows in the context of end-user piracy that if copyright protection is weak then only "customized software" will be developed. Alternatively, in the presence of strong copyright protection both "customized" and "packaged" software will be developed.

However, in the above mentioned literature, enforcement is assumed to be costless and hence is not accounted in the social welfare. Further, copyright protection policy is not determined endogenously and hence, the subgame perfect regime whether to allow or deter copying cannot be identified. So the qualification of their

[5] That is, the copying cost is distributed along a non-decreasing density function. See Novos and Waldman (1984), Proposition 2.
[6] The conclusions in the long-run analysis are more relevant to the issue of incentive to innovate. See Bae and Choi (2006), Proposition 7.

policy implications may be less conclusive. Our paper seeks to overcome these shortcomings.

The main results of our paper are as follows. Increased copyright protection via increased government monitoring effort unambiguously improves the incentive to innovate in the equilibrium *accommodating*- and *aggressive-strategies*. The next set of results concerns the equilibrium monitoring strategy and the consequent market structures. One, it is possible that even the inclusion of the cost of innovation may not result in monitoring by the government as the socially optimal policy. Two, monitoring may be the socially optimal policy but it may not be sufficient to prevent the pirate's entry. This occurs when the accommodating strategy is the subgame perfect equilibrium. Three, monitoring may be the socially optimal outcome and piracy is deterred when the aggressive strategy is the subgame perfect equilibrium.

These results follow from the ambiguity of the properties of the social welfare function with respect to the monitoring rate for the equilibrium *accommodating* and *aggressive* strategies. The ambiguity is due to the following reason. An increase in the monitoring rate increases the product quality which in turn raises the consumer surplus. However, an increase in the monitoring rate also raises the price which in turn reduces the consumer surplus. Thus an increase in the monitoring rate have two opposing effects on the consumer surplus and therefore, on the social welfare function.

Banerjee (2006a, 2006 b) addresses the issue of commercial piracy but not its effect on the incentive to innovate. He shows that no copyright protection is the unique social optimal solution. However, an external factor like special interest lobbying by the monopolist may result in monitoring as the socially optimal outcome but prevention of piracy is not guaranteed.[7] Our paper demonstrates that when the phase of innovation is incorporated, monitoring may emerge as the equilibrium outcome without any external factors like lobbying. However, piracy may or may not be deterred in equilibrium.

This paper is arranged as follows. In sections 2 and 3 we discuss the model and analyze the equilibrium accommodating and aggressive strategies. Sections 4 and 5 contain the government's optimal social welfare policy and some discussions about the penalty. In section 6 we provide the concluding remarks.

2. THE MODEL

We consider four types of agents: the consumers, the monopolist, a pirate who illegally reproduces and sells licensed software, and the government which is responsible for monitoring and penalizing the pirate. We begin our analysis by describing the monopoly situation in the absence of piracy.

There is a continuum of consumers indexed by $\theta, \theta \in [0,1]$. θ is assumed to follow a uniform distribution. We assume there is no resale market for used software. Each consumer is assumed to purchase only one unit of the software. The utility of a type θ consumer is,

$$U(\theta) = \begin{cases} \theta Q - p_m & \text{if the consumer buys the software,} \\ 0 & \text{if the consumer does not buy.} \end{cases} \tag{1}$$

θ is the consumer's valuation of the software, Q is the quality of the software, and p_m is the price of one unit of the software charged by the monopolist. Thus, in the model, consumers differ from one another on the basis of their valuation of the software.

[7] In Banerjee (2006a) the government is solely responsible for monitoring and penalizing the pirate. In Banerjee (2006b) the monopolist is responsible for monitoring and bears the monitoring cost, and the government sets the penalty.

θ_m is the marginal consumer who is indifferent between buying and not buying:

$$U(\theta_m) = Q\theta_m - p_m = 0 \Rightarrow \theta_m = \frac{p_m}{Q}. \qquad (2)$$

In the absence of piracy, the monopolist faces the demand function,

$$D_m(p_m) = \int_{\theta_m}^{1} d\theta = 1 - \frac{p_m}{Q}. \qquad (3)$$

Let $F(Q)$ be the fixed cost of developing a software of quality Q.[8] We assume $F'(Q) > 0$ and $F''(Q) > 0$ for $Q > 0$, and $F'(Q) = 0$ for $Q = 0$. The cost of replicating the software after it has been developed is assumed to be zero. So the monopolist's profit is; $\pi_m = p_m D_m - F(Q)$. The monopolist chooses a price and quality that maximizes its profit. The equilibrium monopoly results are,

$$F'(Q^*) = \frac{1}{4},\ p_m^* = \frac{Q^*}{2}, \text{ and } \pi_m^* = \frac{Q^*}{4} - F(Q^*). \qquad (4)$$

Now, suppose that a commercial pirate exists in the market. The game played between the government, the monopolist, the pirate, and the consumers is specified in extensive form as follows.

Stage 1: The government chooses a monitoring rate α and a penalty G.

Stage 2: The monopolist chooses a price p_m and quality Q.

Stage 3: The pirate observes the monopolist's strategy, and decides to enter or not. If it enters then it chooses a price p_c.

Stage 4: The consumers make their purchase decision.

Let us discuss the behavior of each of the agents in the model. The government only works through the supply side in controlling piracy. Users do not

[8] In the copyright literature, it is also called the "cost of expression" which does not depend on the amount of goods produced. See Landes and Posner (1989).

face the risk of prosecution from the use of pirated software. The government is responsible for monitoring and penalizing the pirate. Let α and G be the monitoring rate and the penalty. The pirate pays the penalty G to the government if his illegal operation is detected. Let $c(\alpha)$ be the cost of monitoring. We assume $c(0) = 0, c'(\alpha) > 0, c'(0) = 0, c''(\alpha) > 0.$

The government chooses α and G to maximize domestic social-welfare subject to a balanced budget constraint.[9] This assumption allows us to treat the monitoring rate as the government's sole choice variable and the penalty is determined following the balanced budget rule. In Section 5 we relax the balanced assumption and discuss its implications. Let R be the net expected revenue of the government from its anti-piracy policy.

$$R = \alpha G - c(\alpha). \tag{5}$$

The balanced budget constraint means $R = 0$. This implies that the penalty equals the average cost of monitoring:

$$G = \frac{c(\alpha)}{\alpha}, \text{ for } \alpha > 0\,. \tag{6}$$

In the absence of monitoring, the penalty is irrelevant. So we assume $G = 0$ if $\alpha = 0$. G is an increasing function of α. By assumption, the marginal cost of monitoring increases with monitoring. So the average cost of monitoring also increases with monitoring.

The pirated software is assumed to be an inferior substitute of the original software.[10] Let qQ be the quality of the pirated software, $q \in (0,1)$, q is given

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[9] Banerjee (2003, 2006a) also assumes the balanced budget rule. Banerjee (2006a) discuss in detail the implications of relaxing this assumption. Qiu (2006) also assumes that the government is responsible for collecting the penalty which is given exogenously because he does not consider costly monitoring.

[10] Banerjee (2006a) mention that the inferior quality of the illegal software can be viewed as the present discounted value of future updates that are available at a lower price and only come with the purchase of the legitimate software. The qualitative difference is intended to capture these aspects and is

exogenously and is common knowledge.[11] The qualitative difference between the original and the pirated software arises because the support benefits and the full warranty that are included with the purchase of the original software do not come with the purchase of the pirated software. We also assume that the pirate's marginal cost of duplicating is zero.

Following Banerjee (2006a, 2006b), the behavior of a type-θ consumer is as follows. One, he only buys the original software. Two, he buys the pirated software if it is available, which occurs with probability $(1-\alpha)$, otherwise he buys the original software. Three, he only buys the pirated software subject to its availability. Four, he buys nothing. So the utility of a type-θ consumer is,

$$U(\theta) = \begin{cases} \theta Q - p_m, & \text{if the user buys the original software,} \\ \alpha(\theta Q - p_m) + (1-\alpha)(q\theta Q - p_c), & \text{if the user buys the original or the pirated software,} \\ (1-\alpha)(q\theta Q - p_c), & \text{if the user buys the pirated software or nothing,} \\ 0, & \text{if the consumer buys nothing.} \end{cases} \tag{7}$$

$q\theta Q$ is the consumer's effective valuation of the pirated software.

There are three marginal consumers, θ_1, θ_2, and, θ_3. θ_1 is the marginal consumer who is indifferent between only buying the original software and buying the original software only if the pirated one is not available. θ_2 is the marginal consumer who is indifferent between buying the original software if the pirated one is not available and only buying the pirated software subject to its availability. θ_3 is the marginal consumer who is indifferent between buying the pirated software subject to its availability and not buying. Hence

assumed to be common knowledge, which allows the monopolist to interact strategically with the pirate. Banerjee (2003), Besen and Kirby (1989), Takeyama (1994) also assume that originals and copies are imperfect substitutes.

[11] We set this bound to ensure that the profits are not indeterminate.

$$\theta_1 Q - p_m = (1-\alpha)(q\theta_1 Q - p_c) + \alpha(\theta_1 Q - p_m) \Rightarrow \theta_1 = \frac{p_m - p_c}{(1-q)Q},$$

$$(1-\alpha)(q\theta_2 Q - p_c) + \alpha(\theta_2 Q - p_m) = (1-\alpha)(q\theta_2 Q - p_c) \Rightarrow \theta_2 = \frac{p_m}{Q}, \qquad (8)$$

$$(1-\alpha)(q\theta_3 Q - p_c) = 0 \Rightarrow \theta_3 = \frac{p_c}{qQ}.$$

We assume that $1 > \theta_1$ and $\theta_2 > \theta_3$. The condition, $1 > \theta_1$, which means that some consumers buy the original product, implies $p_m < p_c + (1-q)Q$. The conditions $\theta_1 > \theta_2$ and $\theta_2 > \theta_3$, which imply that $p_c < qp_m$, means that some consumers buy the pirated good. If $\theta_2 > \theta_3$ then $\theta_1 > \theta_2$ also holds. So the assumption that $1 > \theta_1$ and $\theta_2 > \theta_3$ becomes $1 > \theta_1 > \theta_2 > \theta_3$.[12]

The demand faced by the monopolist consists of users who buy the original software and those who only buy the original software if the pirated one is not available, which occurs with probability α. This characterizes the demand for the original software when the pirate enters the market. If the pirate does not enter, then both groups buy the original software. There is a demand for the pirated software only if the pirate enters the market and is not detected, which occurs with probability $(1-\alpha)$. The monopolist's and pirate's demand functions are,

$$D_m(p_m, p_c, Q, \alpha) = \begin{cases} (1-\theta_1) + \alpha(\theta_1 - \theta_2) = 1 - \dfrac{p_m - p_c}{(1-q)Q} + \alpha\dfrac{qp_m - p_c}{(1-q)Q}, & \text{the pirate enters,} \\ 1 - \theta_2 = 1 - \dfrac{p_m}{Q}, & \text{if the pirate does not enter.} \end{cases}$$

$$D_c(p_m, p_c, Q, \alpha) = \begin{cases} (1-\alpha)(\theta_1 - \theta_3) = \dfrac{(1-\alpha)(qp_m - p_c)}{q(1-q)Q}, & \text{if the pirate enters,} \\ 0, & \text{if the pirate does not enter.} \end{cases}$$

(9)

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[12] The condition $p_c < qp_m$ can be rewritten as $\frac{p_c}{q} < p_m$ where $\frac{p_c}{q}$ is the effective price of pirated software. So the assumption $p_c < qp_m$ means that the price of the legitimate software exceeds the effective price of the pirated product.

We assume that the market for software is quite large and is not fully covered, i.e., $D_m(p_m, p_c, \alpha) + D_c(p_m, p_c, \alpha) < 1$. The consumer surplus is

$$CS = \begin{cases} \int_{\theta_1}^{1} (\theta Q - p_m) d\theta + \alpha \int_{\theta_2}^{\theta_1} (\theta Q - p_m) d\theta + (1-\alpha) \int_{\theta_3}^{\theta_1} (q\theta Q - p_c) d\theta, & \text{if the pirate enters,} \\ \int_{\theta_2}^{1} (\theta Q - p_m) d\theta, & \text{if the pirate does not enter.} \end{cases} \tag{10}$$

A firm remains in the market only if it is making positive profit. The monopolist's and pirate's profits are,

$$\pi_m(p_m, p_c, Q, \alpha) = \begin{cases} p_m - \frac{p_m^2 - p_m p_c}{(1-q)Q} + \frac{\alpha(qp_m^2 - p_m p_c)}{(1-q)Q} - F(Q), & \text{if the pirate enters,} \\ p_m(1 - \frac{p_m}{Q}) - F(Q), & \text{if the pirate does not enter.} \end{cases}$$

$$\pi_c(p_m, p_c, Q, \alpha) = \begin{cases} \frac{(1-\alpha)(qp_m p_c - p_c^2)}{q(1-q)Q} - \alpha G, & \text{if the pirate enters,} \\ 0 & \text{if the pirate does not enter.} \end{cases} \tag{11}$$

The monopolist chooses either an *accommodating* (*ac*) strategy or an *aggressive* (*ag*) strategy. In the case of the *ac-strategy* the monopolist behaves as a leader and chooses a combination of price and quality that maximizes profit assuming that the pirate may enter the market. So the *ac-strategy* plays no role in eliminating the possibility of the pirate's entry. It is only the monitoring rate that can prevent the pirate's entry. The *ag-strategy* is a limit price strategy such that it is not profitable for the pirate to enter the market. In this case, the monopolist plays a strategic role by eliminating the possibility of a pirate's entry.

Social-welfare (*SW*) is the sum of the monopolist's and pirate's profits, the consumer surplus and the government's net revenue, that is,

$$SW = \pi_m + (1-\alpha) p_c D_c - \alpha G + CS + \alpha G - c(\alpha) = \pi_m + (1-\alpha) p_c D_c + CS - c(\alpha)$$

$$= \pi_m + \pi_c + CS\,. \tag{12a}$$

This is because $c(\alpha) = \alpha G$. So $(1-\alpha) p_c D_c - c(\alpha) = (1-\alpha) p_c D_c - \alpha G = \pi_c$. If the pirate does not enter then the social-welfare function is

$$SW = \pi_m + CS - c(\alpha) \tag{12b}$$

We solve the equilibrium of the game by using the method of backward induction. In view of equation (11), the reaction function of the pirate is,

$$p_c = \frac{qp_m}{2}. \tag{13}$$

3. EQUILIBRIUM ACCOMMODATING AND AGGRESSIVE STRATEGIES

In this section we discuss the equilibrium accommodating and aggressive strategies.

3.1 EQUILIBRIUM ACCOMMODATING STRATEGY

In the *ac-subgame* the monopolist's price and the quality strategy allows the pirate's entry. If the government detects the pirate's illegal operations, which occurs with probability α , he pays a penalty G to the government. Substituting the pirate's reaction function, (13) into the monopolist's profit function,

$\pi_m(p_m, p_c, Q, \alpha) = p_m - \dfrac{p_m{}^2 - p_m p_c}{(1-q)Q} + \dfrac{\alpha(qp_m{}^2 - p_m p_c)}{(1-q)Q} - F(Q)$, and equating its first derivatives with respect to p_m and Q to zero gives us the equilibrium *ac-strategy*. The results are summarized in Proposition 1.

Proposition 1. *(i) The monopolist's equilibrium ac-strategy is* $p_m^{ac*}(\alpha) = \dfrac{(1-q)Q^{ac*}}{2-q-\alpha q}$ *where* Q^{ac*} *satisfies* $F'(Q^{ac*}) = \dfrac{1-q}{2(2-q-\alpha q)}$. *The monopolist's and pirate's equilibrium profits are,* $\pi_m^{ac*}(\alpha) = \dfrac{(1-q)Q^{ac*}}{2(2-q-\alpha q)} - F(Q^{ac*})$ *and*

$$\pi_c^{ac*}(\alpha) = \frac{q(1-\alpha)(1-q)Q^{ac*}}{4(2-q-\alpha q)^2} - \alpha G = \frac{q(1-\alpha)(1-q)Q^{ac*}}{4(2-q-\alpha q)^2} - c(\alpha).$$

(ii) The pirate does not enter if $\overline{\alpha} \le \alpha \le 1$, *where* $\overline{\alpha}$ *satisfies* $\pi_c^{ac*}(\overline{\alpha}) = 0$.

(iii) The monopolist's product quality, price and profit for the equilibrium ac-strategy are monotonically increasing in the monitoring rate. The equilibrium profit is convex in the monitoring rate in the interval, $\alpha \in [0,1]$.

(iv) $Q^{ac*}(\alpha) \le Q^{*}$, $\pi_m^{ac*}(\alpha) \le \pi_m^{*}$, *and the monopoly outcome is restored at* $\alpha = 1$.

Proposition 1 implies that an increase in the monitoring rate increases the product quality which is lower than that in the monopoly case. Intuitively, an increase in the monitoring rate decreases the likelihood of the pirate's entry hence the results for the equilibrium accommodating strategy converges to the monopoly outcome.

From part (ii) of Proposition 1 we know that the pirate cannot enter if $\bar{\alpha} \le \alpha \le 1$. Therefore, we get two consumer surplus and social welfare functions for $\alpha \in [0, \bar{\alpha})$ when the pirate enters and for $\alpha \in [\bar{\alpha}, 1]$ when he does not enter. The consumer surplus and social welfare functions for the equilibrium *ac-strategy* are,

$$CS^{ac}(\alpha) = \begin{cases} \dfrac{Q^{ac*}}{2} - \dfrac{7(1-q)Q^{ac*}}{8(2-q-\alpha q)} + \dfrac{(1-q)^2 Q^{ac*}}{4(2-q-\alpha q)^2}, \text{ for } 0 \le \alpha < \bar{\alpha}, \\ \dfrac{Q^{ac*}}{2} - \dfrac{(1-q)Q^{ac*}}{(2-q-\alpha q)} + \dfrac{(1-q)^2 Q^{ac*}}{2(2-q-\alpha q)^2}, \quad \text{for } \bar{\alpha} \le \alpha \le 1. \end{cases} \tag{14}$$

and,

$$SW^{ac}(\alpha) = \begin{cases} SW_1^{ac}(\alpha) = \dfrac{Q^{ac*}}{2} - \dfrac{(1-q)Q^{ac*}}{8(2-q-\alpha q)} - \dfrac{(1-q)^2 Q^{ac*}}{4(2-q-\alpha q)^2} - F(Q^{ac*}) - c(\alpha), \text{for } 0 \le \alpha < \bar{\alpha}, \\ SW_2^{ac}(\alpha) = \dfrac{Q^{ac*}}{2} - \dfrac{(1-q)^2 Q^{ac*}}{2(2-q-\alpha q)^2} - F(Q^{ac*}), \qquad \text{for } \bar{\alpha} \le \alpha \le 1. \end{cases} \tag{15}$$

3.2 EQUILIBRIUM AGGRESSIVE STRATEGY

The monopolist's *ag-strategy* is a limit price strategy such that it is not profitable for the pirate to enter the market. Substitution of the pirate's reaction function in its profit function yields

$$\pi_c(p_m, Q, \alpha) = \frac{(1-\alpha)qp_m{}^2}{4(1-q)Q} - c(\alpha). \tag{16}$$

The pirate does not enter if $\pi_c(p_m, Q, \alpha) = \frac{(1-\alpha)qp_m{}^2}{4(1-q)Q} - c(\alpha) \le 0$, which is,

$$\frac{p_m^2}{Q} \le \frac{4(1-q)c(\alpha)}{q(1-\alpha)}. \tag{17}$$

Let $p_m^{ag*}(\alpha)$ be the equilibrium *ag-strategy* price. The results are summarized in Proposition 2 and the proof is given in the Appendix.

Proposition 2. *(i) The equilibrium ag-strategy is the limit price* $p_m^{ag*}(\alpha)$ *and quality,* $Q^{ag*}(\alpha)$ *where* $p_m^{ag*}(\alpha) = \min\left(\sqrt{\frac{4(1-q)c(\alpha)Q^{ag*}}{q(1-\alpha)}}, \frac{Q^*}{2}\right)$ *and* $Q^{ag*}(\alpha)$ *satisfies*

$F'(Q^{ag*}) = \sqrt{\frac{(1-q)c(\alpha)}{(1-\alpha)qQ^{ag*}}}$. *The monopolist's equilibrium profit is,*

$$\pi_m^{ag*}(\alpha) = \begin{cases} \sqrt{\frac{4(1-q)c(\alpha)Q^{ag*}(\alpha)}{q(1-\alpha)}} - \frac{4(1-q)c(\alpha)}{q(1-\alpha)} - F(Q^{ag*}(\alpha)), & \text{for } 0 \le \alpha \le \alpha_{\max}, \\ \frac{Q^*}{4} - F(Q^*), & \text{for } \alpha_{\max} \le \alpha \le 1. \end{cases}$$

(ii) The monopolist's product quality, price and profit for the equilibrium ag-strategy are monotonically increasing the in the monitoring rate up to $\alpha_{\max}$.

(iii) $Q^{ag*}(\alpha) \le Q^*$, $\pi_m^{ac*}(\alpha) \le \pi^*_m$, *and the monopoly outcome is restored at* $\alpha \ge \alpha_{\max}$.

At, $\alpha = \alpha_{\max}$, where $\alpha_{\max}$ satisfies $\frac{c(\alpha_{\max})}{1-\alpha_{\max}} = \frac{qQ^{ag*}(\alpha_{\max})}{16(1-q)}$, $0 < q < 1$, the entry-deterring limit price is the monopoly price, which is $\frac{Q^*}{2}$. For monitoring rates above the critical level, $\alpha_{\max}$, there is no reason to choose a price more than $\frac{Q^*}{2}$, since that lowers profit and has no effect on entry. For monitoring rates below $\alpha_{\max}$ the entry-deterring limit price is less than the monopoly price. When there is no monitoring, the equilibrium limit price is zero which is obtained by substituting $\alpha = 0$ in $p_m^{ag*}(\alpha) = \sqrt{\frac{4(1-q)c(\alpha)Q^{ag*}}{q(1-\alpha)}}$. At $\alpha = 0$, $F'(Q^{ag*}) = \sqrt{\frac{(1-q)c(\alpha)}{(1-\alpha)qQ^{ag*}}} = 0$, hence $Q^{ag*}(\alpha = 0) = 0$ by assumption. Therefore, $\pi_m^{ag*}(\alpha = 0) = 0$.

The consumer surplus and the social-welfare functions for the equilibrium *ag-strategy* are

$$CS^{ag}(\alpha) = \begin{cases} \dfrac{Q^{ag*}(\alpha)}{2} - \sqrt{\dfrac{4(1-q)c(\alpha)Q^{ag*}(\alpha)}{q(1-\alpha)}} + \dfrac{2(1-q)c(\alpha)}{q(1-\alpha)}, & \text{for } 0 \le \alpha \le \alpha_{\max}; \\ \dfrac{Q^*}{8}, & \text{for } \alpha_{\max} \le \alpha \le 1. \end{cases} \tag{18}$$

$$SW^{ag}(\alpha) = \begin{cases} \dfrac{Q^{ag*}(\alpha)}{2} - \dfrac{2(1-q)c(\alpha)}{q(1-\alpha)} - F(Q^{ag*}(\alpha)) - c(\alpha), & \text{for } 0 \le \alpha \le \alpha_{\max}; \\ \dfrac{3}{8}Q^* - F(Q^*) - c(\alpha), & \text{for } \alpha_{\max} \le \alpha \le 1. \end{cases} \tag{19}$$

3.3 COMPARATIVE STATIC ANALYSIS

Using the results from Propositions 1 and 2 we compare the comparative static analysis of the monopolist's profits and qualities for the *ac-* and *ag-strategies* with respect to the monitoring rate. The results, which are summarized in Proposition 3, will help us to determine the subgame perfect equilibrium strategies and the relevant outcomes for the socially optimal monitoring rates that are discussed in the next section. The proof of Proposition 3 is given in the Appendix.

Proposition 3. *(i) There exists a unique monitoring rate* α_1, $\alpha_1 \in (0, \alpha_{\max})$ *at which* $\pi_m^{ac*}(\alpha_1) = \pi_m^{ag*}(\alpha_1)$. $\pi_m^{ac*}(\alpha) \ge \pi_m^{ag*}(\alpha)$ *for* $0 \le \alpha \le \alpha_1$ *and* $\pi_m^{ag*}(\alpha) \ge \pi_m^{ac*}(\alpha)$ *for* $\alpha_1 \le \alpha \le \alpha_{\max}$.

(ii) The no-piracy monitoring rate, $\overline{\alpha}$ *, for the equilibrium ac-strategy satisfies the condition.* $\overline{\alpha} \in (\alpha_1, \alpha_{\max})$.

Proposition 3 implies that the single-crossing property is satisfied. A diagrammatic representation of Proposition 3 is provided in Figure 1.

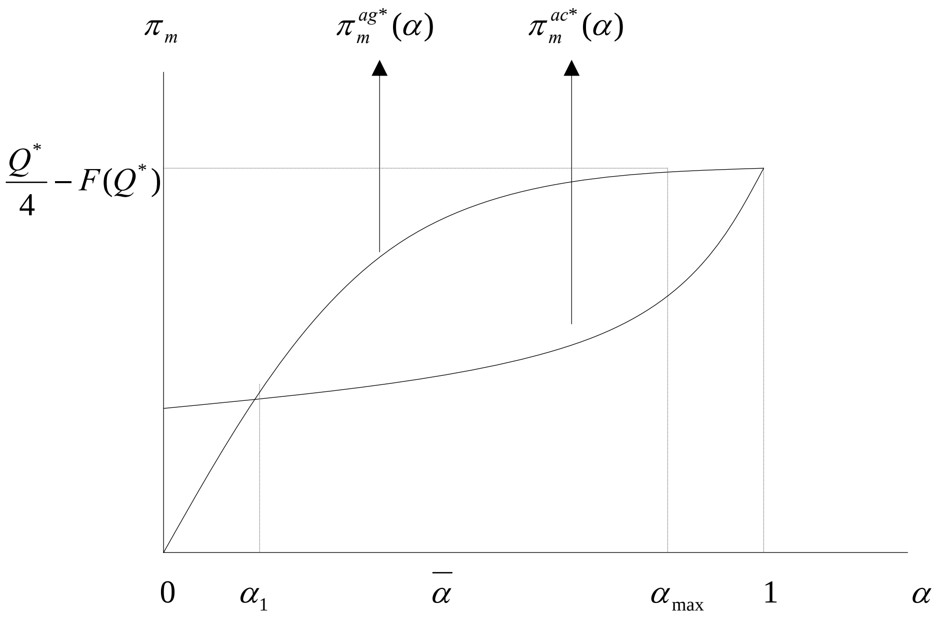

Figure 1

Figure 1 shows that the relevant range of monitoring rate that we need to consider for the rest of the analysis is $\alpha \in [0, \alpha_{\max}]$. This is because, for $\alpha \geq \alpha_1$, the equilibrium *ag-strategy* is weakly dominant, and hence is credible. Increasing α beyond $\alpha_{\max}$ does not change profit or consumer surplus, because the monopoly results are restored in the interval $\alpha \in [\alpha_{\max}, 1]$, but the cost of monitoring, which is a deadweight loss, increases. Therefore, we need to consider the social welfare function corresponding to the equilibrium *ac-strategy* in the interval $\alpha \in [0, \alpha_1)$ and the social welfare function corresponding to the equilibrium *ag-strategy* in the interval $\alpha \in [\alpha_1, \alpha_{\max}]$. So the relevant social welfare functions are,

$$SW_1^{ac}(\alpha) = \frac{Q^{ac*}}{2} - \frac{(1-q)\ Q^{ac*}}{8(2-q-\alpha q)} - \frac{(1-q)^2}{4(2-q-\alpha q)^2} - F(Q^{ac*}) - c(\alpha),\ \text{for}\ \alpha \in [0, \alpha_1),$$

and $SW_1^{ag}(\alpha) = \frac{Q^{ag*}}{2} - \frac{2(1-q)c(\alpha)}{q(1-\alpha)} - F(Q^{ag*}) - c(\alpha)$ for $\alpha \in [\alpha_1, \alpha_{\max}]$.

4 SOCIAL WELFARE ANALYSIS

The government seeks the monitoring rate that maximizes social welfare. The penalty is determined using the balanced budget rule. Let α^{ac*} and α^{ag*} be the monitoring rates that maximizes

$$SW_1^{ac}(\alpha) = \frac{Q^{ac*}}{2} - \frac{(1-q)\ Q^{ac*}}{8(2-q-\alpha q)} - \frac{(1-q)^2}{4(2-q-\alpha q)^2} - F(Q^{ac*}) - c(\alpha),\ \text{for}\ \alpha \in [0, \alpha_1),$$

and $SW_1^{ag}(\alpha) = \frac{Q^{ag*}}{2} - \frac{2(1-q)c(\alpha)}{q(1-\alpha)} - F(Q^{ag*}) - c(\alpha)$ for $\alpha \in [\alpha_1, \alpha_{\max}]$. Let α^* be the socially optimal monitoring rate. The results are summarized in Proposition 4. The proof of Proposition 4 (i) is given in the Appendix. We discuss the proof of Proposition 4 (ii) in the main text because it is instructive.

Proposition 4. *(i) The monitoring rates that maximizes*

$$SW_1^{ac}(\alpha) = \frac{Q^{ac*}}{2} - \frac{(1-q)\ Q^{ac*}}{8(2-q-\alpha q)} - \frac{(1-q)^2}{4(2-q-\alpha q)^2} - F(Q^{ac*}) - c(\alpha),\ \textit{for}\ \alpha \in [0, \alpha_1),$$

and $SW_1^{ag}(\alpha) = \frac{Q^{ag*}}{2} - \frac{2(1-q)c(\alpha)}{q(1-\alpha)} - F(Q^{ag*}) - c(\alpha)$ *for* $\alpha \in [\alpha_1, \alpha_{\max}]$*, satisfies*

$\alpha^{ac*} \in [0, \alpha_1)$ *and* $\alpha^{ag*} \in [\alpha_1, \alpha_{\max}]$.

(ii) The socially optimal monitoring rates and the monopolist's subgame perfect strategies are: (1) $\alpha^* = \alpha^{ac*}$ *if* $SW_1^{ac}(\alpha^{ac*}) > SW_1^{ag}(\alpha^{ag*})$ *and ac-strategy is the subgame perfect equilibrium; (2)* $\alpha^* = \alpha^{ag*}$ *if* $SW_1^{ag}(\alpha^{ag*}) > SW_1^{ac}(\alpha^{ac*})$ *and ag-strategy is the subgame perfect equilibrium.*

The proof of Proposition 4 (i) follows from the ambiguity of the signs of $SW_1^{ac'}(\alpha)$ and $SW_1^{ag'}(\alpha)$. An increase in the monitoring rate increases the product quality which in turn increases the consumer surplus. However, the increase in the monitoring rate increases the price which results in a fall in the consumer surplus. Therefore, the overall effect of increased monitoring on consumer surplus is ambiguous causing the effect of increased monitoring on the social welfare function to be ambiguous as well.[13] Enhanced copyright protection may be a welfare-worsening scenario when the heterogeneity of consumers is such that price-sensitive consumers opt out of the legal market more than those who are lured by higher quality.

Proposition 4(ii) follows from the fact that the government chooses the monitoring rate that yields the highest social welfare. If $SW_1^{ac}(\alpha^{ac*}) > SW_1^{ag}(\alpha^{ag*})$, then the socially optimal monitoring rate is $\alpha^* = \alpha^{ac*} \in [0, \alpha_1)$. From Figure 1 we see that in this range of monitoring rate the accommodating strategy is dominant, and hence, is the subgame perfect equilibrium. So there is piracy in equilibrium. Alternately, if $SW_1^{ag}(\alpha^{ag*}) > SW_1^{ac}(\alpha^{ac*})$, then the socially optimal monitoring rate is $\alpha^* = \alpha^{ag*} \in [\alpha_1, \alpha_{\max}]$. In this case the aggressive strategy is weakly dominant and hence is the subgame perfect equilibrium. Consequently, piracy is deterred in equilibrium.

The implications of Proposition 4 are as follows. One, it is possible that even the inclusion of the cost of innovation may not result in monitoring by the government as the socially optimal policy. This will be the case if $\alpha^* = \alpha^{ac*} = 0$. Two, monitoring

[13] The ambiguity in consumer surplus is also present in Novos and Waldman (1984) and Bae and Choi (2006). Both papers notice that how the shifting of consumers from buying pirates to originals (demand switch) plays out is a key in drawing welfare implication. It is worth noting that the cost of enforcement and equilibrium market structure is not accounted for in these studies.

may be the socially optimal policy but it may not be sufficient to prevent the pirate's entry. This will be the case if $\alpha^{*}=\alpha^{ac*}$ and $0<\alpha^{ac*}<\alpha_1$ in which case the accommodating strategy is the subgame perfect equilibrium. Three, monitoring may be the socially optimal outcome and piracy is deterred if $\alpha^{*}=\alpha^{ag*}$. In this case the aggressive strategy is the subgame perfect equilibrium. The monopoly outcome is restored if $\alpha^{*}=\alpha^{ag*}=\alpha_{\max}$. The socially optimal rate depends on the properties of the social welfare functions $SW_1^{ac}(\alpha)$ and $SW_1^{ag}(\alpha)$ which in turn depends on the sensitivity of consumers to price and quality changes, the cost of developing the product (*F(Q)*), and the cost of administering copyright laws ($c(\alpha)$).

Thus the socially optimal monitoring rate endogenously determines the equilibrium market structure. That is, the socially optimal monitoring policy determines whether commercial piracy exists in equilibrium or not.

5. IMPLICATIONS OF RELAXING THE BALANCED BUDGET ASSUMPTION

In this section we relax the balanced budget assumption and assume that the penalty *G* is institutionally given and the government's expected net revenue is positive that is, $R=\alpha G-c(\alpha)>0$. The social welfare function is,

$$SW=\pi_m+(1-\alpha)p_cD_c-\alpha G+CS+\alpha G-c(\alpha)=\pi_m+(1-\alpha)p_cD_c+CS-c(\alpha).$$

Let us begin with the *ac-strategy*. In this case all the results summarized in Proposition 1 hold except the expression for the pirate's profit which becomes,

$\pi_c^{ac*}(\alpha)=\dfrac{q(1-\alpha)(1-q)Q^{ac*}}{4(2-q-\alpha q)^2}-\alpha G.$ The new level of monitoring beyond which the pirate cannot enter the market is $\overline{\overline{\alpha}}$ that satisfies

$\pi_c^{ac*}(\overline{\overline{\alpha}}) = \dfrac{q(1-\overline{\overline{\alpha}})(1-q)Q^{ac*}}{4(2-q-\overline{\overline{\alpha}}q)^2} - \overline{\overline{\alpha}}G = 0.$ The equilibrium prices, monopolist's equilibrium profit and quality are independent of *G*. The consumer surplus and the social welfare functions are the same as in equations (14) and (15) except that $\overline{\alpha}$ is replaced by $\overline{\overline{\alpha}}$. This is because the penalty from the pirate to the government is a transfer payment and hence does not appear in the social welfare functions. Hence, the properties of these functions are retained.

Let us now consider the *ag-strategy*. The equilibrium limit price and the quality are $p_m^{ag*}(\alpha, G) = \min\left(\sqrt{\dfrac{4(1-q)\alpha G Q^{ag*}}{q(1-\alpha)}}, \dfrac{Q^*}{2}\right)$ and $Q^{ag*}(\alpha, G)$ that satisfies

$F'(Q^{ag*}) = \sqrt{\dfrac{(1-q)\alpha G}{(1-\alpha)qQ^{ag*}}}$. The monopolist's equilibrium profit is,

$$\pi_m^{ag*}(\alpha, G) = \begin{cases} \sqrt{\dfrac{4(1-q)\alpha G Q^{ag*}}{q(1-\alpha)}} - \dfrac{4(1-q)\alpha G}{q(1-\alpha)} - F(Q^{ag*}), & \text{for } 0 \le \alpha \le \alpha_{\max}, \\ \dfrac{Q^*}{4} - F(Q^*), & \text{for } \alpha_{\max} \le \alpha \le 1. \end{cases}$$

We see that in contrary to the *ac-strategy*, in case of the *ag-strategy* the monopolist's equilibrium price, quality, and profit depends on *G*. The results of the comparative static analysis of π_m^{ag*} with respect to *G* are summarized in Proposition 5 and the proof is in the Appendix.

Proposition 5. *Given* α*,* $\pi_m^{ag*}(\alpha, G) = \sqrt{\dfrac{4(1-q)\alpha G Q^{ag*}}{q(1-\alpha)}} - \dfrac{4(1-q)\alpha G}{q(1-\alpha)} - F(Q^{ag*})$ *is monotonically increasing in G and reaches a maximum at* $G_{\max}$*, where* $G_{\max} = \dfrac{qQ^{ag*}(1-\alpha)}{16\alpha(1-q)}$*. At* $G_{\max} = \dfrac{qQ^{ag*}(1-\alpha)}{16\alpha(1-q)}$ *the monopoly outcome is restored,* *hence for* $G > G_{\max}$*, the monopolist retains its monopoly price and quality levels. The monitoring level at which the monopoly outcome is restored is decreasing in the penalty, that is,* $\alpha_{\max}$ *is decreasing in G.*

Figure 2 provides a diagrammatic representation of Proposition 5 and the comparative static analysis of π_m^{ac*} and π_m^{ag*} with respect to *G*.

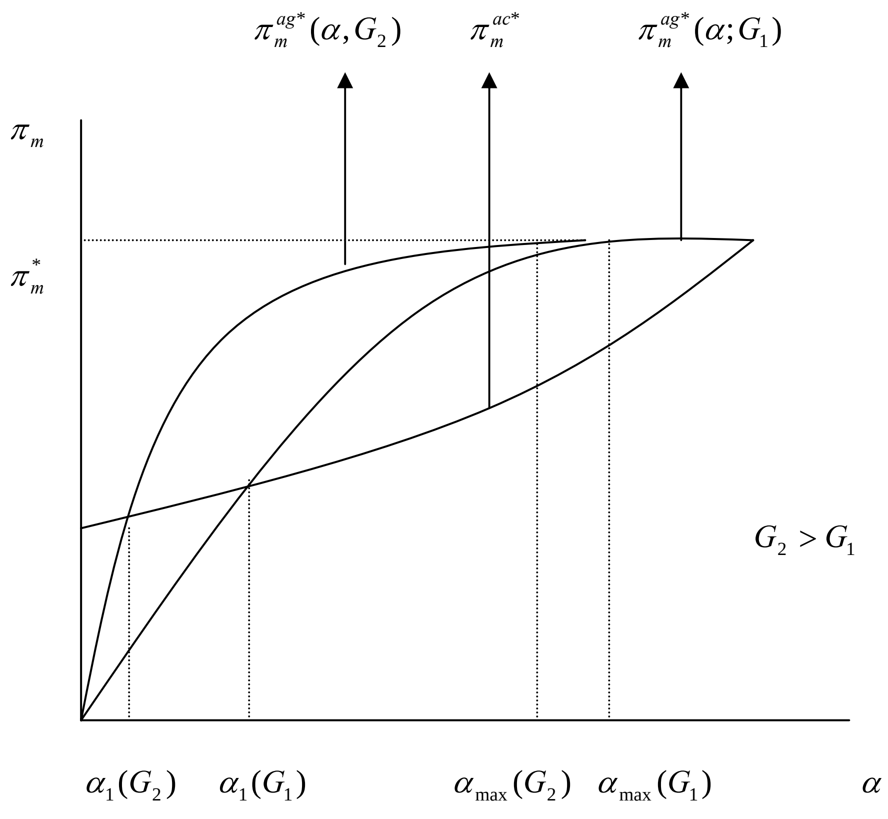

Figure 2

Figure 2 shows that π_m^{ac*} is unaffected by the penalty. However, an increase in the penalty causes an upward shift in π_m^{ag*}, hence $\alpha_{\max}$ and α_1 falls. This implies that the relevant range of monitoring rate for the social welfare function corresponding to the equilibrium *ac-strategy* shrinks while that for the equilibrium *ag-strategy* increases if there is an increase in the penalty.

The social welfare function for the equilibrium *ag-strategy* is,

$$SW_1^{ag}(\alpha, G) = \frac{Q^{ag*}}{2} - \frac{2(1-q)\alpha G}{q(1-\alpha)} - F(Q^{ag*}) - c(\alpha) \text{ for } \alpha \in [\alpha_1, \alpha_{\max}].$$

$\frac{dSW_1^{ag}(\alpha, G)}{dG} = (\frac{1}{2} - F'(Q^{ag*}))\frac{dQ^{ag*}}{dG} - \frac{2(1-q)\alpha}{q(1-\alpha)}$. The sign of $\frac{dSW_1^{ag}(\alpha, G)}{dG}$ is ambiguous because the first term is positive since at the highest quality level which is Q^*, $F'(Q^*) = \frac{1}{4}$, and $\frac{dQ^{ag*}}{dG} > 0$ for $0 \le Q^{ag*} \le Q^*$. Intuitively, an increase in the penalty raises the monopolist's equilibrium price, product quality and profit. However, the effect on consumer surplus is ambiguous because an increase in the penalty raises the price and the quality which in turn has opposing effects on consumer surplus. The properties of the social welfare function for the equilibrium *ag-strategy* with respect to the monitoring rate are the same as mentioned in the case with the balanced budget rule.

Since the social welfare function for the equilibrium *ac-strategy* is unchanged and the properties of the social welfare function for the equilibrium *ag-strategy* are the same as in the case with the balanced budget rule hence, the results for the social welfare maximizing monitoring rate is the same as in Proposition 4. That is, monitoring may or may not be the socially optimal solution and even if monitoring is socially optimal, prevention of piracy is not guaranteed. We have shown that the results summarized in Proposition 4 are robust to a general form enforcement budget rules other than the balanced budget one.[14]

6. CONCLUSIONS

To achieve accurate assessments of the efficacy of copyright protection, we have developed a framework that weaves together the strategic interaction among the

[14] Unfortunately, the comparative static analysis of the socially optimal monitoring rate with respect to the penalty is algebraically intractable.

copyright enforcing government, the innovating and price-setting monopolist, a potential entrant who practices commercial piracy, and a set of taste-heterogeneous consumers.

The existing literature on commercial piracy and the general literature on end-user piracy treat cost of developing the product as a sunk cost and often assume costless copyright protection, with focus on pricing strategies and anti-piracy policies. By incorporating the innovation phase, a costly enforcement scheme, and using an entry-deterring framework, this paper has demonstrated that increased copyright protection via increased government monitoring effort unambiguously improves the incentive to innovate in the equilibrium *accommodating*- and *aggressive-strategies*. Nevertheless, except in the limiting case where the monopoly outcome is restored, the commercial piracy, deterred or otherwise, does dampen the incentive to innovate.

Regarding the socially optimal monitoring rates and the resultant equilibrium market structures, we find that even the inclusion of the cost of innovation may not result in monitoring by the government as the socially optimal policy. Consequently, the accommodating strategy is the subgame perfect equilibrium and there is piracy. Monitoring may be the socially optimal policy but its intensity may not be sufficient to prevent the pirate's entry. In this case also the accommodating strategy is the subgame perfect equilibrium. Alternatively, monitoring may be the socially optimal outcome and piracy is deterred when the aggressive strategy is the subgame perfect equilibrium.

APPENDIX

Proof of Proposition 1: (i) Substituting $p_c = \frac{qp_m}{2}$ in the monopolist's profit function and applying the first order conditions yield;.

$$\frac{d\pi_m(p_m, Q, \alpha)}{dp_m} = 1 - \frac{(2-q-\alpha q)p_m}{(1-q)Q} = 0,$$
$$\frac{d\pi_m(p_m, Q, \alpha)}{dQ} = \frac{(1-q)}{2(2-q-\alpha q)} - F'(Q) = 0.$$

It follows that $p_m^{ac*}(\alpha) = \frac{(1-q)Q^{ac*}}{2-q-\alpha q}$ where Q^{ac*} satisfies $F'(Q^{ac*}) = \frac{1-q}{2(2-q-\alpha q)}$.

Consequently, $p_c^{ac*}(\alpha) = \frac{q(1-q)Q^{ac*}}{2(2-q-\alpha q)}$. For the second order condition we construct the Hessian determinant which is

$$H = \begin{vmatrix} \frac{d^2\pi_m}{dp_m{}^2} & \frac{d^2\pi_m}{dp_m dQ} \\ \frac{d^2\pi_m}{dQdp_m} & \frac{d^2\pi_m}{dQ^2} \end{vmatrix} = \begin{vmatrix} \frac{-(2-q-\alpha q)}{(1-q)Q^{ac*}} & \frac{(2-q-\alpha q)p_m^{ac*}}{(1-q)Q^{ac*2}} \\ \frac{(2-q-\alpha q)p_m^{ac*}}{(1-q)Q^{ac*2}} & \frac{-(2-q-\alpha q)p_m^{ac*2}}{(1-q)Q^{ac*}} - F''(Q^{ac*}) \end{vmatrix}.$$

$H_1 = \frac{-(2-q-\alpha q)}{(1-q)Q^{ac*}} < 0$ and $H_2 = \frac{(2-q-\alpha q)}{(1-q)Q^{ac*}} F''(Q^{ac*}) > 0$. Hence the second order conditions of maximization are satisfied. Substituting $p_m^{ac*}(\alpha)$ and $p_c^{ac*}(\alpha)$ in the monopolist's and the pirate's profit functions gives us

$\pi_m^{ac*}(\alpha) = \frac{(1-q)Q^{ac*}}{2(2-q-\alpha q)} - F(Q^{ac*})$ and

$$\pi_c^{ac*}(\alpha) = \frac{q(1-\alpha)(1-q)Q^{ac*}}{4(2-q-\alpha q)^2} - \alpha G = \frac{q(1-\alpha)(1-q)Q^{ac*}}{4(2-q-\alpha q)^2} - c(\alpha).$$

(ii) In $\pi_c^{ac*}(\alpha) = \frac{q(1-\alpha)(1-q)Q^{ac*}}{4(2-q-\alpha q)^2} - c(\alpha)$ the first expression is the pirate's expected revenue. Let $TR_c^{ac*}(\alpha) = \frac{q(1-\alpha)(1-q)Q^{ac*}}{4(2-q-\alpha q)^2}$. By assumption $-c(\alpha)$ is monotonically decreasing in α. Let us study the properties of $TR_c^{ac*}(\alpha)$.

$$\pi_c^{ac*}(\alpha = 1) = -c(\alpha = 1), TR_c^{ac*}(\alpha = 1) = 0, \ \pi_c^{ac*}(\alpha = 0) = TR_c^{ac*}(\alpha = 0) = \frac{q(1-q)Q^{ac*}}{4(2-q)^2}.$$

So by the intermediate value theorem, there exists a α, say $\overline{\alpha}$, at which

$\pi_c^{ac*}(\alpha) = \dfrac{q(1-\alpha)(1-q)Q^{ac*}}{4(2-q-\alpha q)^2} - c(\alpha) = 0$. We want to prove that for $\overline{\alpha} \le \alpha \le 1$,

$\pi_c^{ac*}(\alpha) \le 0$. Note that

$\dfrac{dTR_c^{ac*}}{d\alpha} = \dfrac{q(1-q)}{4}[\dfrac{(1-\alpha)(2-q-\alpha q)Q'^{ac*} + (-2+3q-\alpha q)Q^{ac*}}{(2-q-\alpha q)^3}]$, hence

$\dfrac{dTR_c^{ac*}}{d\alpha}|_{\alpha=1} = \dfrac{q(-2+2q)Q^*}{8(1-q)^2} < 0$. We then discuss two cases,

(1) If $\dfrac{dTR_c^{ac*}}{d\alpha}|_{\alpha=0} \le 0$, then in this case $TR_c^{ac*}(\alpha)$ is maximized at $\alpha = 0$. This means that $TR_c^{ac*}(\alpha)$ is monotonically decreasing in α, which means that

$\pi_c^{ac*}(\alpha) = \dfrac{q(1-\alpha)(1-q)}{4(2-q-\alpha q)^2} - c(\alpha)$ is also monotonically decreasing in α. So for

$\overline{\alpha} \le \alpha \le 1$, $\pi_c^{ac*}(\alpha) \le 0$.

(2) If $\dfrac{dTR_c^{ac*}}{d\alpha}|_{\alpha=0} > 0$, $TR_c^{ac*}(\alpha)$ is maximized at an interior solution, say, $\alpha_{TR}^* \in (0,1)$. So $TR_c^{ac*}(\alpha)$ monotonically increases up to α_{TR}^* and then monotonically decreases. Therefore, depending upon the slope of $c(\alpha)$, we can conclude that either $\pi_c^{ac*}(\alpha)$ is monotonically decreasing or it monotonically increases up to a certain monitoring rate and then monotonically decreases. So there is a unique $\overline{\alpha}$ at which $\pi_c^{ac*}(\alpha) = 0$.

From (1) and (2), we can conclude that for $\overline{\alpha} \le \alpha \le 1$, $\pi_c^{ac*}(\alpha) \le 0$.

(iii) $\dfrac{dQ^{ac*}}{d\alpha} = \dfrac{q(1-q)}{2(2-q-\alpha q)^2 F''(Q^{ac*})} > 0$ since $F''(Q^{ac*}) > 0$.

$\dfrac{dp_m^{ac*}}{d\alpha} = \dfrac{(1-q)}{(2-q-\alpha q)^2}\left((2-q-\alpha q)\dfrac{dQ^{ac*}}{d\alpha} + qQ^{ac*}\right) > 0$., because $\dfrac{dQ^{ac*}}{d\alpha} > 0$.

$\pi_m^{ac*}(\alpha) = p_m^{ac*}(\alpha) - \dfrac{(2-q-\alpha q)p_m^{ac*2}}{2(1-q)Q^{ac*}} - F(Q^{ac*})$. Applying envelope theorem we get

$\dfrac{d\pi_m^{ac*}(\alpha)}{d\alpha} = \dfrac{\partial \pi_m^{ac}}{\partial \alpha}\Bigg|_{p_m = p_m^{ac*}, Q=Q^{ac*}} = \dfrac{qp_m^{ac*2}}{2(1-q)Q^{ac*}} > 0$. Further,

$\dfrac{d^2\pi_m^{ac*}(\alpha)}{d\alpha^2} = \dfrac{qp_m^{ac*}\left(\dfrac{qp_m^{ac*}}{2-q-\alpha q} + \dfrac{dp_m^{ac*}}{d\alpha}\right)}{2(1-q)Q^{ac*}} > 0$.

(iv) $F'(Q^*) - F'(Q^{ac*}) = \dfrac{1}{4} - \dfrac{1-q}{2(2-q-\alpha q)} = \dfrac{q(1-\alpha)}{4(2-q-\alpha q)} > 0$. Since *F'(Q)* is increasing in *Q* by assumption, hence $Q^{ac*}(\alpha) \le Q^*$. Since $\pi_m^{ac*}(\alpha)$ is monotonically increasing in α and $\pi_m^{ac*}(\alpha = 1) = \pi_m^* = \dfrac{Q^*}{4} - F(Q^*)$, because at $\alpha = 1$

$F'(Q^{ac*}) = F(Q^*) = \dfrac{1}{4}$ which means $Q^{ac*}(\alpha = 1) = Q^*$**,** therefore, $\pi_m^{ac*}(\alpha) \le \pi_m^*$.

Q.E.D.

Proof of Proposition 2: (i) The monopolist faces the following constrained profit maximization problem.

$$\max_{p_m, Q} \pi = p_m - \frac{p_m^2}{Q} - F(Q)$$

subject to $g(p_m, Q, \alpha) = \dfrac{p_m^2}{Q} - \dfrac{4(1-q)c(\alpha)}{q(1-\alpha)} \le 0.$

Hence the Lagrangian function of this optimization is

$$L(p_m, Q, \lambda) = p_m - \frac{p_m^2}{Q} - F(Q) - \lambda\left(\frac{p_m^2}{Q} - \frac{4(1-q)c(\alpha)}{q(1-\alpha)}\right).$$

The first order conditions are;

$$\frac{\partial L}{\partial p_m} = 1 - \frac{2p_m}{Q} - \frac{2\lambda p_m}{Q} = 0,$$

$$\frac{\partial L}{\partial Q} = \frac{p_m^2}{Q^2} - F'(Q) + \frac{\lambda p_m^2}{Q^2} = 0,$$

$$\frac{\partial L}{\partial \lambda} = -(\frac{p_m^2}{Q} - \frac{4(1-q)c(\alpha)}{q(1-\alpha)}) = 0.$$

It follows that $p_m^{ag*}(\alpha) = \sqrt{\frac{4(1-q)c(\alpha)Q^{ag*}}{q(1-\alpha)}}$ where Q^{ag*} satisfies

$F'(Q^{ag*}) = \sqrt{\frac{(1-q)c(\alpha)}{q(1-\alpha)Q^{ag*}}}$. For the second order condition we construct the bordered

Hessian which is,

$$H = \begin{vmatrix} 0 & g_{p_m} & g_Q \\ g_{p_m} & L_{p_m p_m} & L_{p_m Q} \\ g_Q & L_{Q p_m} & L_{QQ} \end{vmatrix} = \begin{vmatrix} 0 & \frac{2p_m}{Q} & -\frac{p_m^2}{Q^2} \\ \frac{2p_m}{Q} & \frac{-2(1+\lambda)}{Q} & \frac{2(1+\lambda)p_m}{Q^2} \\ -\frac{p_m^2}{Q^2} & \frac{2(1+\lambda)p_m}{Q^2} & \frac{-2(1+\lambda)p_m^2}{Q^3} - F''(Q) \end{vmatrix}.$$

$H_1 = \frac{-4p_m^{ag*2}}{Q^{ag*2}} < 0$ and $H_2 = \frac{2(1+\lambda)p_m^{ag*4}}{Q^{ag*5}} + \frac{4p_m^{ag*2}F''(Q^{ag*})}{Q^{ag*2}} > 0$ because $F''(Q) > 0$.

Hence the second order conditions of maximization are satisfied.

(ii) Total differentiation of the first order conditions and rearranging terms yield,

$$\frac{dQ^{ag*}}{d\alpha} = \frac{4(1-q)[c'(\alpha)(1-\alpha) + c(\alpha)]Q^{ag*2}}{(1-\alpha)^2 p_m^{ag*2}} > 0,$$

$$\frac{dp_m^{ag*}}{d\alpha} = \frac{4(1-q)[c'(\alpha)(1-\alpha) + c(\alpha)]Q^{ag*}}{(1-\alpha)^2 p_m^{ag*}} > 0.$$

Since $\pi_m^{ag*}(\alpha) = p_m^{ag*}(\alpha) - \frac{p_m^{ag*2}}{Q^{ag*}} - F(Q^{ag*})$, applying the envelope theorem we get

$$\frac{d\pi_m^{ag*}(\alpha)}{d\alpha} = \left.\frac{\partial L}{\partial \alpha}\right|_{p_m = p_m^{ag*}, Q = Q^{ag*}} = \frac{4\lambda(1-q)[c'(\alpha)(1-\alpha) + c(\alpha)]}{(1-\alpha)^2} > 0.$$

(iii) Positivity of the Lagrangian multiplier $\lambda > 0$ implies $p_m^{ag*} < \frac{Q^{ag*}}{2}$. Substituting

$p_m^{ag*}(\alpha) = \sqrt{\frac{4(1-q)c(\alpha)Q^{ag*}}{q(1-\alpha)}}$ this inequality can be rewritten as

$\frac{1}{4} - \sqrt{\frac{(1-q)c(\alpha)}{q(1-\alpha)Q^{ag*}}} > 0$. Since *F'(Q)* is increasing in *Q* by assumption and

$F'(Q^*) - F'(Q^{ag*}) = \frac{1}{4} - \sqrt{\frac{(1-q)c(\alpha)}{q(1-\alpha)Q^{ag*}}} = \frac{q(1-\alpha)}{4(2-q-\alpha q)} > 0$, hence $Q^{ag*}(\alpha) \le Q^*$.

Substituting $\alpha = \alpha_{\max}$ in the expressions for Q^{ag*}, p_m^{ag*}, and π_m^{ag*} we see that the monopoly outcome is restored. *Q.E.D.*

Proof of Proposition 3. (i) From Proposition 1 we know that $\pi_m^{ac*}(\alpha)$ and is monotonically increasing in α, $\alpha \in [0,1]$. Further, $\pi_m^{ac*}(\alpha = 0) > 0$ and $\pi_m^{ac*}(\alpha = 1) = \pi_m^*$. From Proposition 2 we know that $\pi_m^{ag*}(\alpha)$ is also monopolistically increasing in α, $\alpha \in [0, \alpha_{\max}]$. Further, $\pi_m^{ag*}(\alpha = 0) = 0$ and $\pi_m^{ag*}(\alpha = \alpha_{\max}) = \pi_m^*$. Therefore, $\pi_m^{ag*}(\alpha)$ is steeper than $\pi_m^{ac*}(\alpha)$ in the range, $0 \le \alpha \le \alpha_{\max}$, and the single-crossing property is satisfied in the range $0 \le \alpha \le \alpha_{\max}$. Let α_1 be the monitoring rate at which the monopolist's equilibrium profits for the *ac*- and *ag-strategies* are equal. The single-crossing property means that $\pi_m^{ac*}(\alpha) \ge \pi_m^{ag*}(\alpha)$ for $0 \le \alpha \le \alpha_1$ and $\pi_m^{ag*}(\alpha) \ge \pi_m^{ac*}(\alpha)$ for $\alpha_1 \le \alpha \le \alpha_{\max}$.

(ii) Substituting $p_m^{ac*}(\overline{\alpha}) = \frac{(1-q)Q^{ac*}(\overline{\alpha})}{2-q-\overline{\alpha}q}$ into $\pi_c^{ac*}(\alpha)$ we get

$\pi_c^{ac*}(\overline{\alpha}) = \frac{(1-\overline{\alpha})qp_m^{ac*2}}{4(1-q)Q^{ac*}} - c(\overline{\alpha}) = 0$ which on rearrangement yields,

$\frac{(1-q)^2 Q^{ac*}(\overline{\alpha})}{4(2-q-\overline{\alpha}q)^2} = \frac{(1-q)c(\overline{\alpha})Q^{ag*}(\overline{\alpha})}{q(1-\overline{\alpha})Q^{ag*}(\overline{\alpha})}$. Replacing terms with $F'(Q^{ac*}(\overline{\alpha})) = \dfrac{1-q}{2(2-q-\overline{\alpha}q)}$ and $F'(Q^{ag*}(\overline{\alpha})) = \sqrt{\dfrac{(1-q)c(\overline{\alpha})}{(1-\overline{\alpha})qQ^{ag*}(\overline{\alpha})}}$ we have

$F'(Q^{ac*}(\overline{\alpha}))^2 Q^{ac*}(\overline{\alpha}) = F'(Q^{ag*}(\overline{\alpha}))^2 Q^{ag*}(\overline{\alpha})$. By continuity and monotonicity of

$F(Q)$ we know that $Q^{ac*}(\overline{\alpha}) = Q^{ag*}(\overline{\alpha})$. Similarly, $p_m^{ac*}(\overline{\alpha}) = p_m^{ag*}(\overline{\alpha})$. Let

$\overline{Q} = Q^{ac*}(\overline{\alpha}) = Q^{ag*}(\overline{\alpha})$. From Propositions 1 and 2 we know that

$\pi_m^{ac*}(\overline{\alpha}) = \dfrac{(1-q)\overline{Q}}{2(2-q-\overline{\alpha}q)} - F(\overline{Q})$ and

$\pi_m^{ag*}(\overline{\alpha}) = \sqrt{\dfrac{4(1-q)c(\overline{\alpha})\overline{Q}}{q(1-\overline{\alpha})}} - \dfrac{4(1-q)c(\overline{\alpha})}{q(1-\overline{\alpha})} - F(\overline{Q})$. Substituting

$\dfrac{(1-q)^2\overline{Q}}{4(2-q-\overline{\alpha}q)^2} = \dfrac{(1-q)c(\overline{\alpha})}{q(1-\overline{\alpha})}$ we can rewrite the equilibrium profit functions for the

ag-strategy as $\pi_m^{ag*}(\overline{\alpha}) = \dfrac{(1-q)\overline{Q}}{2-q-\overline{\alpha}q} - \dfrac{(1-q)^2\overline{Q}}{(2-q-\overline{\alpha}q)^2} - F(Q_{\overline{\alpha}})$. Then

$\pi_m^{ag*}(\overline{\alpha}) - \pi_m^{ac*}(\overline{\alpha}) = \dfrac{(1-q)\overline{Q}}{2(2-q-\overline{\alpha}q)} - \dfrac{(1-q)^2\overline{Q}}{(2-q-\overline{\alpha}q)^2} = \dfrac{q(1-\overline{\alpha})(1-q)\overline{Q}}{2(2-q-\overline{\alpha}q)^2} > 0$.

Now $\pi_m^{ag*}(\alpha_1) = \pi_m^{ac*}(\alpha_1)$. For $\alpha_1 < \alpha \le \alpha_{\max}$, $\pi_m^{ag*}(\alpha) > \pi_m^{ac*}(\alpha)$. Therefore, $\alpha_1 < \overline{\alpha}$.

Note that at, $\alpha = \alpha_{\max}$, where $\alpha_{\max}$ satisfies $\dfrac{c(\alpha_{\max})}{1-\alpha_{\max}} = \dfrac{qQ^{ag*}(\alpha_{\max})}{16(1-q)}$, $0 < q < 1$, the

entry-deterring limit price and quality converge to the monopoly outcome, i.e.,

$\left(p_m^{ag*}(\alpha_{\max}), Q^{ag*}(\alpha_{\max})\right) = (\dfrac{Q^*}{2}, Q^*)$. Now $Q^{ac*}(\overline{\alpha}) = Q^{ag*}(\overline{\alpha})$ and

$F'(Q^{ag*}(\alpha_{\max})) - F'(Q^{ag*}(\overline{\alpha})) = \dfrac{1}{4} - \dfrac{1-q}{2(2-q-\overline{\alpha}q)} = \dfrac{q(1-\overline{\alpha})}{4(2-q-\overline{\alpha}q)} > 0, \text{for } 0 < \overline{\alpha} < 1$.

Since *F'(Q)* is increasing in *Q* by assumption, hence $Q^{ag*}(\overline{\alpha}) \le Q^{ag*}(\alpha_{\max})$ which in

turn implies $\overline{\alpha} < \alpha_{\max}$. *Q.E.D.*

Proof of Proposition 4: (i) $SW_1^{ac'}(\alpha) =$

$$\frac{[q(1-q)(2-3q+\alpha q)]Q^{ac*}(\alpha)}{8(2-q-\alpha q)^3}+[\frac{1}{2}-\frac{(1-q)(12-7q-5\alpha q)}{8(2-q-\alpha q)^2}]\frac{dQ^{ac*}(\alpha)}{d\alpha}-\mathrm{c}'(\alpha),$$

for $0 \le \alpha < \alpha_1$.

$$SW_1^{ag\,\prime}(\alpha)=\frac{2(1-q)}{q}\cdot\frac{c'(\alpha)(1-\alpha)+c(\alpha)}{(1-\alpha)^2}+\left(\frac{1}{2}-F'(Q^{ag*})\right)\frac{dQ^{ag*}(\alpha)}{d\alpha}-\mathrm{c}'(\alpha),$$

for $\alpha_1 \le \alpha < \alpha_{\max}$.

Now $SW_1^{ac\,\prime}(\alpha)\begin{smallmatrix}>\\=\\<\end{smallmatrix}0$ and $SW_1^{ag\,\prime}(\alpha)\begin{smallmatrix}>\\=\\<\end{smallmatrix}0$. Consequently, $\alpha^{ac*}\in[0,\alpha_1)$ and

$\alpha^{ag*}\in[\alpha_1,\alpha_{\max}]$. *Q.E.D.*

Proof of Proposition 5: $\frac{d\pi_m^{ag*}}{dG}=\frac{1}{2}\sqrt{\frac{(1-q)\alpha Q^{ag*}}{Gq(1-\alpha)}}-\frac{4\alpha(1-q)}{q(1-\alpha)}$. Equating this to 0

yields $G_{\max}=\frac{qQ^{ag*}(1-\alpha)}{16\alpha(1-q)}$. At $G_{\max}=\frac{qQ^{ag*}(1-\alpha)}{16\alpha(1-q)}$, $F'(Q^{ag*})=\sqrt{\frac{(1-q)\alpha G}{(1-\alpha)qQ^{ag*}}}=\frac{1}{4}$.

That is, at $G_{\max}=\frac{qQ^{ag*}(1-\alpha)}{16\alpha(1-q)}$, $F'(Q^{ag*})=F'(Q^*)=\frac{1}{4}\Rightarrow Q^{ag*}=Q^*$, therefore,

$p_m^{ag*}=p_m^*=\frac{Q^*}{2}$. For any *G*, $\alpha_{\max}$ satisfies $\frac{\alpha_{\max}G}{1-\alpha_{\max}}=\frac{qQ^*}{16(1-q)}$. We are interested in finding the properties of the locus of $\alpha_{\max}$ and *G* such that the monopoly outcome is restored, that is $Q^{ag*}=Q^*$. Taking the total differential of $\frac{\alpha_{\max}G}{1-\alpha_{\max}}=\frac{qQ^*}{16(1-q)}$ with respect to $\alpha_{\max}$ and *G* yields, $\frac{d\alpha_{\max}}{dG}=-\frac{qQ^*(1-\alpha_{\max})^2}{16G^2(1-q)}<0$. *Q.E.D.*

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