Copying and Demand for Originals

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Abstract

Information goods are consumption goods that derive a large part of their value from their data content. Understanding markets for information goods has become more urgent with the rise of the Internet marketplace. Because computers excel at copying data, such goods are vulnerable to competition from their own copies.

Previous literature has modeled the exchange of information goods, but most stripped important characteristics of information goods from the analysis.

This paper attempts to address some previously neglected aspects of music, an information good. An innovation of this paper is the flavor, a subtype of a spatial good, similar to vertical differentiation in IO literature. In this terminology, "originals" (CDs) and "copies" (mp3s) are flavors of music.

The paper argues that an important but often overlooked distinction between music originals and copies is that originals are more valuable (higher quality) than copies. The paper also models the stock and habit effects of music, and inquires whether a drop in the price of copies can increase the demand for originals over the short or long run, as is claimed by defenders of copying. Optimal pricing of originals in the face of diminishing price of copies is also investigated.

In addition, the introduction of a possible third flavor, a data-only original, is analyzed. An optimal data-only original would increase the total revenues from originals and diminish the demand for copies.

The results indicate that it is indeed possible but unlikely that a decrease in price of copies leads to an increase in demand for originals. A more likely result is that the demand for originals drops, but the total demand for music increases. Adjusting the price of originals can diminish the drop in demand and revenue. However, producers can mitigate their losses by introducing a third flavor. The paper documents the attempts of the producers of originals to find the optimal price and quality level of the third flavor.

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1 Introduction

In the past couple years, there has been a change in the way people get music. A de facto standard for easy and free distribution of music on the Internet emerged. The standard is, of course, the on-line sharing of music encoded with MPEG-1[1] layer III audio compression technology, which produces music files commonly known as mp3s. Napster, the most famous on-line firm where such sharing occurs, provided the critical mass

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[1] MPEG stands for Moving Picture Experts Group, and it works on standards for the coding of moving pictures and associated audio.

necessary to bring the word mp3 to everyone's consciousness and take the sharing of mp3s from the domain of computer-geeks to the masses. By early 2001 Napster the was shut down by copyright owners. However, the spirit of Napster lives on, and many file sharing protocols and sharing communities have emerged.

Mp3 sharing begun without the backing of any organization or firm. It was the consumers of music that recognized such sharing as a way to get more music and spend less money, and a few entrepreneurial music fans set up sharing protocols and communities. They did this with only themselves in mind, and so a system with no restrictions or monetary costs appeared. This did not please the owners of the copyrights to the shared music. An mp3 is a computer file containing a song, and when it is "shared," an identical copy appears on the computer of the recipient. Unlike the sharing of physical items containing music, where the music can be at only one place at a time, sharing mp3s allows any number of people to simultaneously access the same music. Copyright owners are in the business of being paid for providing copies of their music, so they obviously did not like this characteristic of mp3 sharing. Since it first occurred to copyright owners what was happening, they have been trying to set up an alternative standard. None have very successful thus far.

This paper tries to show that this development is not necessarily detrimental to the copyright owners. To do this, the paper models demand for copies and originals over time, and determine whether decreasing the "price" of copies can increase the demand for originals in the long run. In addition, the paper models the introduction of a legitimate downloadable music format, designed to maximize the revenues to the copyright owners and minimize the spread of mp3s. This analysis is relevant to the movie industry, as well. The widespread sharing of movies is still limited by the lack of necessary bandwidth, but the bandwidth is increasing fast.

For the purposes of this paper, there are two types of copying: commercial copying, such as the manufacture of unauthorized CDs, and "consumer" copying, such the sharing of mp3s on the Internet. The paper deals with consumer copying only, as this is the much more prevalent way of unauthorized copying.

1.1 Structure

The paper is structured as follows: Section 2 contains a review of both non-academic and academic literature on the effect of copying on the demand for originals. Section 3 introduces the model and defends its main characteristics. Section 4 contains the various variations of the model and develops its mathematical properties. Section 5 contains a few examples of the model in action. Section 6 introduces the alternative music format. Section 6 concludes the paper.

2 Literature Review

The effects of Napster and its ilk on sales of music CDs are unknown, but are generally considered to be negative. After all, reasoning goes, why spend $10-$20 on a CD with only one or two good songs when you can download them for free in no time?[2] Some mp3 fans disagree and claim that mp3 sharing makes it easy to find old, no longer sold favorites; to "media" shift the music one already owns on vinyl records or audio cassettes; to preview music before buying, etc., but not to substitute buying new CDs.

There is a vast economic literature dealing with copyright and other intellectual property economics, and an increasing number or papers deal with the copying and sharing of music. A few commercial studies tried to determine if consumers that share music still buy it as well. Here is a brief summary of the relevant literature.

[2] Similar arguments are made for all digital goods. Mulligan (1995) argues that the availability of pornography on the internet will strip the creators of pornography of income (due to unauthorized distribution) and thus drastically reduce the amount of new pornography created. It seems this did not happen.

2.1 Non-Academic Studies

As mp3s became all the buzz and claims surfaced that Internet music piracy is costing the music industry billions of dollars per year, many firms scrambled to be among the first to release some kind of survey related to mp3s and direct some of that attention to themselves.

There are two types of commercial studies: surveys about music expenditures (e.g. SoundScan 2000 and the Fine Report) and tracking of sales of music in record stores near universities (e.g. Jupiter Communications 2000 and the Jay Report). The typical results were that people reported higher expenditures on music after exposure to mp3's, but record stores lost sales. Both types of studies had their critics who dismissed the findings.

A better study was done by PC Data (2000). It tracked 120,000 Internet users with software installed on their computers and found that "new Napster users are just as likely to purchase music at CDNow in the first month after downloading Napster software as they were before becoming members of the file-swapping service. But those consumers' purchases decreased after 90 days, even though they visited on-line music retailers more frequently." Unfortunately, PC Data tracked if people bought from CDNow, but not how much they spent, or if they bought music from anywhere else.

US music sales numbers are provided yearly by the Recording Industry Association of America (RIAA) in their "Consumer Profile" report. In 1999, CD sales (in dollars) were up 18% from the previous year, while singles were down 16%. In 2000, CD sales increased 5%, and singles fell 55%. In 2001, CD sales were down 4%, and singles were down 16%. Also during 2001, sales in North America declined by 4.7%, sales in Europe declined by 0.8%, and sales in Japan fell by 9.4%. Sales of cassettes have been declining for many years, but the decline in the sales of singles started in fairly recently, in 1998. As CD singles and mp3s are close substitutes, more so than full-length CDs and mp3s, it is likely that this is a direct consequence of mp3 sharing.

Some research disputes RIAA's claim that mp3 sharing is the largest cause of sales loss. Moore (2003) looked at the 1999-2002 sales figures for the largest US firms and found that RIAA sales fell similarly to sales of many of the firms. According to Moore (2003), it is entirely possible that the economic weakness is the major cause of the fall in music sales.

To gage the overall popularity of mp3's, the Pew Internet Tracking Report (Graziano and Rainie 2001) comes in handy. It claims that 30 million American adults have (ever?) downloaded music from the Internet, and that their numbers are increasing rapidly (by 40% in the six months before the release of the report).

A more current survey, Ipsos (2003), found that 40 million American adults have downloaded mp3 music in the last 30 days, and that "downloaders believe their actions don't hurt artists." Downloading was particularly prevalent among younger people, with almost half of those surveyed below age 25 have downloaded mp3's in the last month.

2.2 Copying Literature

Since the first appearance of discussion of copyright and copying in economics journals, conventional economists (Arrow, 1962) held that government support is necessary "to promote the progress of science and useful arts" (Article I, Section 8, U.S. Constitution), although there was dissent from the likes of Frase (1966), Breyer (1970) and Hughes (1988). Among the believers, the main topic of interest was the trade-off between short run distortions (underutilization) and long run efficiency[3] (underproduction) and its applications to optimal patents (e.g. Tandon, 1982) and optimal copyright (e.g. Breyer 1970). There is discussion on the technical violation of copyright such as: smuggling in international trade (e.g. Bhagwati and Hansen, 1973) and parallel imports and gray markets (e.g. Gallini and Hollis 1999; Papadopoulos, 2000). Ordover

[3]Long run efficiency is a question only when the supply of creative works or innovations can vary. For a view at an art without copyright protection see Frank (1996). The art in question is chess game scores.

and Willig (1978) and Liebowitz (1985) dwell on the copying of academic journals and the related price discrimination and indirect appropriability. Takeyama (1994) explores the effect of demand network externalities on firm profits, and two Gayer and Shy (2002a,b) papers incorporate unusual network externalities in their analyses of copying and hardware taxation. Landes and Posner (1989) model an author creating works in part based on previous works, and show how increasing copyright protection may have negative supply consequences. Besen and Raskind (1991) is a nice introduction to the law and economics of intellectual property.

As copying technology progresses, it inevitably becomes easier and cheaper for consumers to copy creative works (or generally any partially nonexcludable goods) without compensating their owners. Each time a quantum leap occurs in copying technology, proposals for further restrictions on copying follow, although the content producing industry has yet to suffer major losses from any of them. This happened after the introduction of the Xerox 914 dry paper copier in 1959, the Sony Beta Video Cassette Recorders in 1974, the digital audio tape (DAT) in the 1980s, etc. Quickly following these calls for restrictions on copying come papers theoretically analyzing such restrictions. Novos and Waldman (1984) is one such paper, Johnson (1985) is another. Novos and Waldman (1984) look at a market for a theoretical partially nonexcludable good where a monopolist is the sole supplier, and the consumers either consume one or none of the good. With all consumers valuing the good identically but having different copying costs, they find no social welfare loss due to underutilization, and only tenuous social welfare loss due to underproduction. This differs from most other analyses, and Johnson (1985) claims to have more realistic models with product variety and multiple producers. One successor to Novos and Waldman (1984) is Yoon (2002), who modifies their model to include differences between copies and originals in a quest to understand the optimal level of copyright protection.

Some of the more recent papers have looked at copying empirically. Hui, Png and Cui (2002) and Andres (2002) estimate the effects of income and copyright protection on piracy rates across time and countries. Varian (2000) describes how buying clubs can diminish demand but can support a higher price - a version of indirect appropriability. Takeyama (2002) develops an asymmetric information model where copying reveals an unobservable quality, and producers have an incentive to allow copying in order to signal that their product is of high quality. Boldrin and Levine (2002a,b) develop a model of perfectly competitive innovation and use it to argue that intellectual property protection is not necessary for the optimal functioning of innovation markets. However, what they describe resembles more the functioning of the legitimate recording market today, with copyright protections, rather than the unprotected Internet copying of mp3s.

2.3 Johnson's (1985) "marginal cost only" model

Following some earlier product differentiation papers like Salop (1979), Johnson (1985) represents the variety of creative works[4] by locations on a circle. Each creative work is defined by a position on this circle, and each author is free to choose that position. Consumers are also located on the circle, and their positions represent their tastes for the creative works. The position can be thought as the consumer's "ideal point." A consumer i with fixed preferences μ_i will consume a creative work j with price p_j if

$$\mu_i \geq p_j + \psi_{ij},$$

where ψ_{ij} represents the distance between the consumer and the creative work along the circle. This consumption decision function indicates that consumers prefer cheaper creative work that are closer to

[4] Johnson (1985) refers to these goods as "creative works" which can be either "originals" or copies. The creative works that are offered for sale by their authors or copyright holders are referred to as originals, although they are not originals in the sense of unique paintings or sculpture. Perhaps a better use of language would be to call these official copies or authorized copies. The consumers then buy these originals, or make their own copies, which are not official or authorized. Either way, they consume that creative work.

their tastes. It is linear, with a definite cutoff point: the consumer will not consume (enjoy, appreciate, etc.) creative works whose total cost (price plus distance) is greater than some number representing that consumer's overall level of love for the arts (or whatever the creative works in question are). In case copying is "allowed," it is thought of as a household copying technology, with copy costs ω_i, composed of a time wage and other costs, that will determine if consumer i consumes copies or original creative works.

This demand function is well behaved: an increase in price decreases demand. Other determinants of demand are copying costs and distance between the consumer's ideal point and the location of the creative work.

Figure 1 (identical to Figure 2 in Johnson (1985), with additional on-graph explanation) shows the copy/buy decision of consumers at a point of the circle for some distribution of μ and ω. The decision is for some creative work with a fixed p and ψ. Those to the right of the $\omega = p$ line will only entertain the option to buy or not buy, while those to the left will choose between copying or not copying only. Those above the $\mu = p + \psi$ line will rather buy than not buy, while those above the $\mu = \omega + \psi$ diagonal line will rather copy than not copy. Thus we have the regions that will buy, copy, or not consume this particular creative work. It is clear that fewer consumers will consume more distant works, and that a consumer will either buy or copy all consumed works, but never mix buying and copying.

Johnson (1985) then goes to conclude that an increase in copying has uncertain effects on the price the author-monopolist[5] charges for her creative work (due to the possible increase of the slope of the demand function), but that revenue would certainly decrease. Furthermore, as Johnson (1985) views authors' marginal cost as 0, any switch from buying to copying by a consumer actually decreases her contribution to total consumer surplus, so an increase in copying may have a negative effect on social welfare from the consumer's point of view as well. In the long run, social welfare will depend on the numbers of people switching from buying to copying and the elasticity of supply.

3 A Different Copying Model

My copying model follows Johnson (1985). However, it adds to it the fact that copies are not identical to the originals - thus are not perfect substitutes; the fact that there is always some copying going on - thus changes in parameters can increase or decrease copying (i.e. consumption of copies) and consumption of originals but cannot (on the margin) fully destroy the incentive to produce creative works; and the fact that marginal utility of creative works is not necessarily constant, but that it depends on a person's exposure to music.

Incorporating these differences into Johnson's (1985) model makes it more applicable to the market for music CDs and mp3s, as is explained in this section. However, to leave the possibility that such modelling can apply to other creative works, and to preserve compatibility with previous copying literature, the language of the general copying model remains.

3.1 Where do Mp3s Come From?

One aspect of Internet copying that the model doesn't take into consideration is how consumers find copies of originals they don't own. Finding mp3s on the Internet and downloading them is a time-intensive process, while making them from CDs is much less so. It seems that sharing of mp3s on the Internet is not explainable by simple economics. Those that share get no payment or other consideration, their reputation is not affected, and their actions are illegal. However, the sharing does not take many resources other than bandwidth: one can share and listen to mp3s at the same time. Once a (desirable) file is shared by someone, many copies appear extremely quickly. For these reasons, the model will consider mp3s to be just "there," available for

[5] Is a copyright holder a monopolist? Kitch (2000) disagrees.

the taking to anyone with time, a computer and an Internet connection. This seems to be a reasonable approximation of reality.

Sharing of mp3s provides little or no tangible benefits. In the early days, FTP site owners would require uploads before allowing downloads, and thus sharing included a payment in kind. Currently, few sharing programs enforce any kind of reciprocity sharing. Those that do, prioritize downloads from people that have many shared files or have many uploads. This method is imperfect, as one could share many fake popular files yet appear to be a prodigious sharer. Some sharing programs allow sharing of partially downloaded files, increasing the supply of downloadable files.

According to some studies, a large *percentage* of music downloaders do not share any files themselves, but a large enough *number* of users share many files, although it is illegal to do so. Adar and Huberman (2000), show that 70% of gnutella (a sharing network) users share no files (they are colloquially known as leechers). There are few large sharers - the top 1% of sharers shared 40% of the network's total, the top 5% shared 71%, and the top 20% share 95% of the total. According to Wired magazine (Sept. 2001), there are more than one million computers with a thousand or more mp3's (10% of the total sharing community, they estimate) that occasionally connect to the various sharing networks. The average sharer is connected for a fairly short time (typical duration is 12 minutes). The typical content is music (49%), porn (23%), other video (21%), and some software and other image files.

The fact that the majority of the network users do not share with others, but want to take what others offer, is an expected behavior in a reputation free economy. The mystery is the others, those that do share with no obvious payment, who nevertheless are in sufficient numbers to keep the sharing system going strong.

It is easier to explain the actions of the creators of sharing software like Napster (now defunct), Napigator, OpenNap, Gnutella, Limewire, Bearshare, Grokster, KaZaA, KaZaA Lite, Morpheus, eDonkey, eMule, Shareaza and many others. The software is provided for free, but it is used to serve ads, sell t-shirts, ask for donations, and create prestige for the software makers.

[THIS SECTION IS AWKWARD]

3.2 Quality Difference

The idea that copies are not identical to originals is not disputed. In fact, Johnson (1985) proposal for extensions of his copying model contain the same mathematical structure for differentiating originals and copies as the one used in this paper. Quality difference helps resolve a problem in the copying literature. Johnson (1985) and other papers do not consider a possibility that Liebowitz (1985) raises: copying costs may be smaller than the marginal cost of the producers. Johnson (1985) dismisses this by saying that if this were the case, the producer would employ the copier's production technology. This cannot happen when the product of the two technologies are different. However, it clearly does apply to music CDs and mp3s. Two Gayer and Shy (2002a,b) papers and Takeyama (2002) also use quality difference between copies and originals in their analyses.

An original full-length music CD bought in a retail store is a sturdy plastic disk containing a collection of songs 30 to 80 minutes in length, typically comes in a protective box, with a booklet containing information on songs and artwork, and is sold for $10-$20. A CD single is an audio CD containing between two and five songs, typically several version of a popular song and a less known song. Single CDs rarely come with artwork of linear notes, and cost about $3-$10. The music stored on the CD is called CD audio (CD-A or CD-DA occasionally, as in digital) and, in accordance to the "red book" standard that defines it, uses a sample rate of 44.1 kHz[6], with each sample 16 bits in size, and two samples taken at a time for stereo sound.

[6] McGoodwin (2000) explains that a "[s]ampling theorem of Harry Nyquist 1928 (and his precursors) states that a continuous band-limited signal may be exactly represented (without any loss of data) by amplitude samples made at a sampling frequency S equal to twice the highest signal frequency component. In other words, the Nyquist (half-sampling) frequency S/2 is the highest freq. that can be accurately represented by a sampling frequency of S." Since the human ear can hear up to about 20

Therefore, each second of CD audio takes 1,411,200 bits of data. A CD can contain 74 minutes of audio, but it can go up to 80 minutes with some minor modifications to the red book standard. CDs can contain additional data that can be used with a computer, such as music videos, screen savers, and interviews with the musicians. Some CD makers took the opposite approach, and made CDs that are incompatible with (some) computer CD-ROMs and therefore (not much) harder to copy.

A CD is quite small: 2mm thick and 120mm in diameter and sturdy too (it could last 100 years with proper care). In addition CD audio also has excellent sound reproduction properties. As McGoodwin (2000) explains:

> Standard CD-A (PCM 16-bit per stereo channel 44.1 kHz sampling) yields in typical players a frequency response from 5 Hz to 20 kHz of ± 0.2 dB. Dynamic range exceeds 100 dB, signal/noise ratio (S/N) exceeds 100 dB, and channel separation exceeds 100 dB at 1 kHz. Harmonic distortion at 1 kHz is $< 0.002\%$, and wow and flutter are essentially unmeasurable. With digital filtering, phase shifts are less than 0.5 degrees. Linearity is within 0.5 dB at -90 dB.

An mp3 is a computer file that contains audio that has been compressed with methods that take into consideration peoples perception of sound. These "perceptual coding" techniques are based on psychoacoustic principles and attempt to decrease the storage space (and thus bandwidth when the data moves) required for sound of a specific quality as perceived by the listeners. This is achieved by reducing and eliminating inaudible or masked portions of the original sound. Mp3 compression was developed by the Fraunhofer IIS-A, and standardized in 1993 under the name MPEG-1 layer III. There are several commercially used psychoacoustic data reduction technologies, and mp3 is just one of them. It became widely used mostly because it was the first compression technology that packed near-CD quality sound into file sizes that common computers could handle, and Fraunhofer IIS-A did not enforce licensing or request royalty payments from the makers of commercial or free codecs (COmpression-DECompression programs). This changed in September 1999, and has caused the disappearance of small-scale mp3 software makers, while increasing interest in alternative codecs such as Ogg Vorbis.

A "standard-issue" mp3 file contains one song encoded into a constant 128 kbps (kilo bits per second) stream, which is about 11 times less data than CD audio. The sound quality has been described as near-CD, but it is impossible to measure sound quality of perceptually coded sound with conventional tools. Instead, listening tests are used, where the compressed sound is compared to its source. Typical acoustic problems reported for perceptually coded sound can be diminished by using higher bitrate encoding. Mp3 can go as high as 320 kbps, where it achieves a compression of only about 4.4 times. This is not so great as there are lossless compressions[7] that result in files about half the size of the CD source. The bitrate can also be made variable, which can achieve a constant perceived level of compression for music of varying (psychoacoustic) intensity.

Storing music on a computer has its benefits, and this is likely the reason that mp3s became as popular as they are: music becomes computer files that can be quickly copied, traded, and aggregated into virtual jukeboxes. Music was copied and traded before the advent of mp3s as well, with audio cassettes exchanged among friends. Home-use jukeboxes were available as well, with CD players that can hold from 3 to several hundred CDs at once. But cassettes had to be recorded in real time, while to copy a music file can take less than a second. Home-use jukeboxes are expensive single-purpose machines, and are hard to program due to their complicated user interfaces. Computers with hundreds or thousands of mp3s allow users to organize,

kHz, a sampling rate of at least 40 kHz is needed to reproduce all of the audible sound. CDs use a sampling rate of 44.1 kHz, DATs (Digital Audio Tape) use 48 kHz, etc. Schulzrinne (1996) explains why the CDs use 44.1 kHz exactly.

[7] Lossless compression is compression where no data is lost. Normal computer data compression is lossless, like gzip, but images and sound can be compressed with lossy algorithms, where the information that is considered imperceptible is lost, e.g. jpeg for images or mp3 for sound. A problem with lossy compression in general is that cascading compression can cause severe quality loss. This is less pronounced with good (and well implemented) compression algorithms.

find, sort and play mp3s, with the same technology and interfaces that users are already familiar with.[8] Plus, all this music takes no physical space, as computers are already a part of many households, and are useful for other tasks, as well. A typical \$100 home-use hard disk (HD) of 2003 can hold 120GB of data (in April 2001 \$100 could buy a 20GB HD, and in April 2002 \$100 could buy a 20GB HD). Such a HD can hold 240 average length CDs uncompressed, or 2640 with standard mp3 compression. Even 100 CDs in their cases can take quite a bit of space, and the sheer number can make it hard to find a particular one. As HD capacity increases, so will the amount of music that can be stored on them. Will users migrate to higher bitrate compression or even uncompressed CD audio? Time will tell.

Mp3's are not tied to computers. Many portable players emerged. The smallest ones store the music files on flash memory, and can usually contain only a few hours of music. Slightly larger ones, such as the Apple iPod, store music on a small hard disk, and can store hundreds of hours of music. There are also portable CD players that can read mp3 filled CDs. These are usually much larger than the mp3 players, as they have to accommodate a full size CD. Due to the ease of carrying around a large quantity of mp3 music, it is safe to say that mp3s are more portable than CDs, although they play on fewer devices.

Since CDs are of better sound quality than mp3s, and mp3s are seemingly more useful than CDs, which should be assigned the higher overall quality? The fact that music from CDs can be made into mp3's quickly and easily with any computer equipped with a modern CD-ROM drive and some free software makes CDs the better choice. A CD confers both the benefits of the CD and the benefits of the mp3. The opposite is not the case. Mp3s can be written onto recordable CDs (CD-Rs or CD-RWs), but the sound quality will be lower than the original CDs as the music went through a compression cycle. Also, recordable CDs are physically more fragile than CDs, do not last as long, are susceptible to damage in even moderate sunlight, and lack labels and linear notes. Thus it is clear that CDs are superior to mp3s, and can be assigned a higher overall quality value.

This relationship is not so clear with CD singles. They are likely bought either for the one popular song, or for the alternative versions of the popular song. If they are bought for the popular song, they may act as samples for which the quality and durability of the media is not too important. If they are bought for the alternative versions of the popular song, often more versions are available as mp3s. In either case the qualities that make CDs more valuable than mp3s are less important for CD singles, and it may very well be said that the overall quality level of mp3s is higher than that of CD singles.

3.3 Pervasive Copying

The introduction of mp3 technology and its widespread use is not the first time copying of music has flourished. Remember audio cassettes, those ubiquitous sound containers that hold about 60-90 minutes of music that can be reproduced fairly well? Until they were made obsolete by CD-RW drives and mp3s, cassettes were the medium of choice for casual copiers. To listen to that vinyl record in the car, one had to first record it onto a audio cassette. Later, when CDs replaced records, and cars were not yet equipped with CD players, the same had to be done for CDs. Cheap CD recording technology made it possible to copy CDs without loss of audio quality (but with loss of other desirable characteristics) and mp3s made copying even easier, now without physical access to the source CD. Thus the current development with mp3 sharing is but one (large) step in the ever present decline in copying costs.

At several occasions in the past consumer copying was accused of being the harbinger of the destruction of media companies. The audio cassette was to destroy all record companies, and the video tape was to destroy all TV and movie companies. Needless to say, that didn't happen. The firms adapted and exploited the new technology, making it a significant source of their revenues. Similarly, mp3 music is soon expected to destroy record companies. I suspect that they can survive if they, once again, adapt to the situation and

[8] There are many devices that can take mp3s on the go, either as flash memory, CD-R, or hard disk based players, and more will be developed as demand for them increases.

use the new technology to their advantage. Suppressing the copying of music will not work, as the ability to copy has never before decreased, and will likely not decrease in the future. Instead, selling music that includes value added components that cannot be copied in the near future will likely become the next savior of the record companies.

Consumer copying haunts many other industries. The needlework and sewing pattern publishers have seen their products scanned and traded via email. Similar things happen to cooking recipe publishers and subscriber only websites.

[MORE EXAMPLES]

3.4 Marginal Utility and Exposure to Music

Becker and Stigler (1977) formalized the theory behind the accumulation of the consumption capital (a sort of beneficial addiction), but the concept was not unknown before that. Alfred Marshall had this to say about it in his 1923 book, "Principles of Economics:"

> There is however an implicit condition in this law [of diminishing marginal utility] which should be made clear. It is that we do not suppose time to be allowed for any alterations in the character of the tastes of the man himself. It is therefore no exception to the law that the more good music a man hears, the stronger is his taste for it going to become.

In this spirit, the model in this paper will incorporate past exposure to music as habit, which will influence the current consumption decision.

Some papers have explored this possibility, e.g. Silva and Ramello (2000) argue that home taping allowed consumers with low valuations to become music listeners and become the high valuation listeners that record producers wanted. There is also literature on network externalities (e.g. Takeyama, 1994) where more exposure to a product leads to greater demand, but for different reasons.

A similar but opposite effect is the "stock" effect. The more CDs a person has, the less net utility he derives from a new CD because there are already many others that he can listen to. Because each CD is different, I don't believe this effect is very strong. If the typical CD is listened to only a few times, the stock effect would be weak, but it would be stronger if CDs are listened to over and over. An analogy with books would be that mystery novels do not have any stock effect, as they are a one time read (they might have a strong habit effect). However, once a person has a dictionary and a thesaurus, another one is not needed. To understand the joint stock and habit effects, it is necessary to model both the purchase and the consumption (listening) of music. This way, the time constraint would effectively limit the listening, but not the buying of music.

4 The Model

As inspired by Johnson (1985), the variety in creative works is represented by *locations on a circle*. Each point of the circle represent a unit of a creative work (in the case of music, an album or a song - consult Figure 2 for a sketch). Creative works are dense on the circle, and they are distributed according to some distribution function. Creative works near one another are similar, and they differ from those further away. A consumer is represented by an *ideal point* on this circle, and his preferences for creative works depend on the distance from his ideal point to the location of a creative work along the circle. An obvious assumption is that preferences decrease with distance. Different people have different ideal points, but they all agree to the relative positions of albums on the circle. The people at any ideal point are not necessarily identical.

For each creative work the "quality level" distinguishes the copy from the original. Quality refers here to the physical representation of the creative work, not the intrinsic quality of the creative works.[9] The higher the quality level, the more a consumer likes a creative work. Consumer preferences also depend on the breadth (or variety) of creative works the consumer is exposed to, or habit.

4.1 One Flavor

Generalizing this framework, we can talk about a good that comes in several "flavors," each with different quality and price. A consumer will then consume a certain amount of this good, but will subdivide the consumption by flavors as necessary to receive the maximum utility possible. The following sections develop the consumer's decision process for one flavor.

In terms of CDs, we can say that the consumer will buy all CDs nearer to his ideal point and will stop at some point. This point will be further from the ideal point than in the case with no habit effect, as the consumer will take the beneficial effect of today's consumption on tomorrow's utility into effect. The music that the consumer buys today is different from the music of tomorrow - at each period there is a new generation of music to buy or copy. This isn't quite how it works in the real world, but the model could still be insightful enough.

4.1.1 Utility Fundamentals

When contemplating consumption, a person is not limited in the location of the creative works he buys - he could buy one work at a distance a, and another at a distance b. However, in a continuous model utility must be defined over measurable sets, so the consumer must be restricted to buying sets of creative works, e.g. all works in the $[a, b]$ distance segment (i.e. all works with distance between a and b).

Suppose there is a strictly decreasing function $v : \Re_+ \to \Re_+$ defined over distances from ideal points, with $\lim_{x\to 0} v(x) \leq +\infty$. The utility a person receives from buying all creative works in the $[a, b]$ distance segment is

$$u([a, b]) = \int_a^b v(x)dx.$$

Since creative works are located symmetrically for each person, it makes sense to define utility over any measurable set $A \subseteq \Re_+$, and the assume creative works are consumed at distances in A on both sides of the consumer's ideal point. Utility is then defined as follows

$$u(A) = \int_A v(x)dx.$$

This utility formulation incorporates creative works on both sides of the ideal point (use $v/2$ for one side only). It also assumes a *uniform distribution* of creative works on the circle.

More generally, relaxing the uniform distribution assumption, and locating creative works on the real line instead of a circle, utility could be defined as follows. Let $A \subseteq \Re$ be any measurable set, let g be the distribution function of creative works on the real line, let i be the ideal point of the consumer, and let v be function defined above. Then

$$u(A) = \int_A v(x)g(i + x)dx$$

[9] In fact, such linear creative work differentiation models are unable to judge the intrinsic quality of a creative work, the models can be only categorize it by its "location." Perhaps there is a region where "bad" creative works reside, and people with ideal points in that region can be instantly recognized after one look at their living rooms.

is the utility the person at i gets from buying all creative works in A. As an example, let A be the set of all creative works between the distances a and b ($0 < a < b$). We could write $A = [-b, -a] \cup [a, b]$, so utility would be

$$u(A) = \frac{1}{2}\int_{-b}^{-a} v(x)g(i+x)dx + \frac{1}{2}\int_{a}^{b} v(x)g(i+x)dx.$$

Since this formulation adds another layer of complications, it will not be considered here.

Assume all creative works have a constant price p. (Alternatively, there could be a integrable price function p, but this isn't pursued further in this model.) Then if A is the set of creative works a person is buying, he will pay

$$p(A) = p\int_A dx$$

for it. If money enters linearly into the utility function (i.e. the utility is quasilinear), we can write the consumer's problem as

$$\max_A u(A) - p(A). \tag{1}$$

Proposition 1 *The solution to the consumer's problem defined above is $A = [0, \phi]$ where ϕ is implicitly defined in $v(\phi) = p$. ϕ is the largest distance to a consumed creative work. The problem in equation 1 can be then be simplified to read*

$$\max_\phi \int_0^\phi v(x)dx - p\phi.$$

Proof. Start with $A = [a, b]$, and take first order condition with respect to a and b. Consider the boundary conditions $a = 0$, and $b = a$. The problem is

$$\max_A \int_a^b v(x)dx - p(b-a)$$

$$\text{s.t. } 0 \leq a \leq b$$

and the FOCs are

$$\begin{aligned} -v(a) + p &= 0 \text{ if } a > 0 \\ &\leq 0 \text{ if } a = 0 \end{aligned}$$

and

$$\begin{aligned} v(b) - p &= 0 \text{ if } b > a \\ &\leq 0 \text{ if } b = a. \end{aligned}$$

Assuming that $v(0) > p$, we have $a = 0$ and $b > 0$ (where $v(b) = p$). If $v(0)$ is too small, then $a = b = 0$ is the solution. ■

A similar result can be shown for $A = [a_1, b_1] \cup [a_2, b_2]$, where $a_2 = b_1$ and $b_2 = a_2$ must hold in the nontrivial case.

Yet another way to show that consumption will always start at 0 is to note that

$$\int_0^\epsilon v(x)dx - p\epsilon > \int_a^{a+\epsilon} v(x)dx - p\epsilon$$

for any $a > 0$ and small $\epsilon > 0$.

Therefore, the problem in equation 1 can be simplified to read

$$\max_{\phi} u(\phi),$$

where

$$u(\phi) = \int_0^{\phi} v(x)dx - p\phi.$$

This is an indirect utility function, as part of the consumer's problem has already been optimized out. Figure 3 pictures the v function and $u(\phi)$.

4.1.2 Consumption over Time

Using the indirect utility defined above, we add habit level H and place the decision process in a multiperiod setting. The current habit level influences the current ϕ consumption decision, and the current ϕ influences future habit. Here, habit is assumed beneficial, so that an increase in H increases utility and the current consumption of ϕ.

Again, the utility is quasilinear in money, so all goods other than ϕ are summarized in z. The utility is of the form

$$U(\phi, z, H) = u(\phi, H) + z$$

and the budget constraint is

$$p\phi + z = I.$$

The utility need not be quasilinear, i.e. it could be just additively separable in z, like $U(\phi, z, H) = u(\phi, H) + w(z)$. Such utility would detract, I think, from looking at the behavior of ϕ and H.

From the properties of v we know that the utility function $u(\phi, H)$ is positive, increasing and concave in ϕ ($u_\phi > 0$ and $u_{\phi\phi} < 0$). In addition, assume that it is increasing in H ($u_H > 0$), and that the static demand function for ϕ is well defined for any p ($\lim_{\phi\to\infty} u_\phi \to 0$) and increasing in H , that is $\partial\phi/\partial H > 0$. This last requirement implies that, and that $u_{\phi H} > 0$, and comes from differentiating the demand function $u_\phi = p$ with respect to. H, which results in

$$u_{\phi\phi}\frac{\partial\phi}{\partial H} + u_{\phi H} = 0.$$

Another assumption can be that $u_{HH} < 0$, if necessary for a well behaved solution. To later assign a quality level as a multiplicative constant, u may not cross 0, it must be always either above or below 0. For example, u can be Cobb-Douglass, or another two-good utility function with $u_{12} > 0$.

Repeating this problem over time, with fixed income and prices, and a discount factor $\beta \in (0, 1)$, the question arises about the behavior of H_t, and the existence of a steady state. The infinite sequence problem then, is

$$\max_{\{\phi_t, H_t, z_t\}} \sum_{t=0}^{\infty} \beta^t \left(u(\phi_t, H_t) + z_t\right)$$

$$\begin{aligned} \text{s.t. } I &= p\phi_t + z_t, \text{ and} \\ H_{t+1} &= (1-\delta)H_t + \phi_t, \forall t = 0, 1, ... \end{aligned}$$

The corresponding functional equation (value function) is

$$v(H) = \max_{0\leq\phi} \left\{u(\phi, H) + I - p\phi + \beta v(H')\right\}, \tag{2}$$

(this v is different from the one in fundamentals) and the law of motion for habit is

$$H' = (1 - \delta)H + \phi.$$

The policy function $g(H)$ selects the optimal ϕ, and the function $G(H)$ selects H'.

4.1.3 Model Properties

This is not the standard value function formulation, so v may not have the usual nice properties associated with such problems. This section shows some properties of the function v. Following Stokey and Lucas (1996), several things can be shown about the value function in equation 2.

Equation 2 is generalized by the expression

$$(Tw)(x) = \max_{y \in \Gamma(x)} \{F(x, y) + \beta w(y)\}.$$

This expression defines a functional operator T that maps functions of the form $w : X \to \Re$ to other such functions, with $X \subseteq \Re^l$. The correspondence $\Gamma : X \to X$ defines the feasible set of y's given an $x \in X$. Call a sequence $\Pi(x_0)$ a feasible plan if

$$\Pi(x_0) = \{\{x_t\}_{t=0}^{\infty} : x_{t+1} \in \Gamma(x_t),\ t = 0, 1, ...\}.$$

Define A as the graph of Γ,

$$A = \{(x, y) \in X \times X : y \in \Gamma(x)\},$$

then $F : A \to \Re$.

In the special case described by equation 2, $x \equiv H$, $y \equiv H'$,

$$F(x, y) \equiv u(y - (1 - \delta)x, x) + I - p(y - (1 - \delta)x),$$

$$\Gamma(x) \equiv [(1 - \delta)x, \infty),$$

and

$$X = [0, \infty) \subseteq \Re.$$

The operator T is defined by

$$(Tw)(x) = \max_{y \in \Gamma(x)} \{u(y - (1 - \delta)x, x) + I - p(y - (1 - \delta)x) + \beta w(y)\}.$$

For this T, Assumptions 4.1 and 4.2 in Stokey and Lucas (1996) hold.

Assumption 2 (4.1) $\Gamma(x)$ *is non empty* $\forall x \in X$*.*

Assumption 3 (4.2) *(modified) F is bounded on one side.*

Assumption 4.1. obviously holds, and Assumption 4.2. must hold as well, because can be confirmed as follows. For any $x_0 \in X$, it must hold that $x_t \in X$. Thus it holds that

$$u(x_{t+1} - (1 - \delta)x_t, x_t) + I - p(x_{t+1} - (1 - \delta)x_t) \leq u(I/p, I/(p\delta)) + I$$

and therefore the above limit exists.

These two assumptions guarantee the existence of a function v that satisfies equation 2, and that $v(x)$ is also the value of the sequence problem at the initial point x.

Assumptions 4.3. and 4.4. can also be shown to hold.

Assumption 4 (4.3.) *X is convex, and Γ is nonempty, compact valued and continuous.*

Assumption 5 (4.4.) *The function $F : A \to \Re$ is bounded and continuous and $0 < \beta < 1$.*

Assumption 4.3. obviously holds, and Assumption 4.4. can be confirmed as follows. F is bounded by $u(I/p, I/(p\delta)) + I$ on A. Also, F is obviously continuous, and $0 < \beta < 1$.

These two assumptions guarantee (by Theorem 4.6.) that T has a unique fixed point v in the space of bounded continuous functions on X, and that for any continuous function v_0, $\|T^n v_0 - v\| \leq \beta^n \|v_0 - v\|$ holds $\forall n = 0, 1, 2, ...$ Note that for any $v_n = T^n v_0$ there is a optimal policy function g_n, as well as g for v.

Theorem 4.7. shows that v is strictly increasing, but the two assumptions (4.5. and 4.6.) it uses do not hold in this case. Still, it can be shown that v is strictly increasing.

Proposition 6 *The function v that satisfies equation 2 is strictly increasing.*

Proof. Rewrite T as

$$(Tw)(x) = \max_{0 \leq \phi \leq I/p} \{u(\phi, x) + I - p\phi + \beta w(\phi + (1-\delta)x)\}$$

and note that $u(\phi, x) + I - p\phi$ is strictly increasing in x for any feasible ϕ. Take any nondecreasing w and any $x_1 \in X$, and find its optimal y_1 and by consequence ϕ_1. Then take any $x_2 > x_1$, and find its optimal ϕ_2. Note that ϕ_1 is feasible for x_2, so that

$$u(\phi_1, x_1) + I - p\phi_1 < u(\phi_1, x_2) + I - p\phi_1$$

and

$$\beta w(\phi_1 + (1-\delta)x_1) \leq \beta w(\phi_1 + (1-\delta)x_2)$$

hold (u strictly increasing in H, w nondecreasing). Thus we get

$$(Tw)(x_1) < u(\phi_1, x_2) + I - p\phi_1 + \beta w(\phi_1 + (1-\delta)x_2) \leq (Tw)(x_2)$$

and so $(Tw)(x_1) < (Tw)(x_2)$ for any $x_1 < x_2$, $x_1, x_2 \in X$. Since T maps nondecreasing functions into strictly increasing functions, we know that the fixed point must be a strictly increasing function. Therefore v is strictly increasing. ■

Assumptions 4.7. and 4.8. can also be shown to hold.

Assumption 7 (4.7.) *F is strictly concave.*

Assumption 8 (4.8.) *Γ is convex.*

Define $f(x, y) = u(y - (1-\delta)x, x)$, and note that F is the sum of f and a linear function. f needs to be strictly concave for F to be as well. We know that u is strictly concave in ϕ, H. Create a third parameter, $z = y - (1-\delta)x$, so that $f(x, y) = u(z, x)$. Then it follows that

$$\begin{aligned} f(\theta x_1 + (1-\theta)x_2, \theta y_1 + (1-\theta)y_2) &= u(\theta z_1 + (1-\theta)z_2, \theta x_1 + (1-\theta)x_2) > \\ \theta u(z_1, x_1) + (1-\theta)u(z_2, x_2) &= \theta f(x_1, y_1) + (1-\theta)f(x_2, y_2), \end{aligned}$$

and so F is strictly concave.

Assumption 4.8. is obviously true. Then, by Theorem 4.8., v is strictly concave, and, by Theorem 4.9., the policy function g_n converges pointwise to g (and uniformly if F is continuously differentiable).

Assumption 4.9. (F is continuously differentiable in the interior of A) holds and thus Theorem 4.11. applies. It says that v is continuously differentiable at all interior points x ($x \in \operatorname{int} X$) where $y = g(x)$ is also interior.

4.1.4 Steady State and Linearization

Since v is continuously differentiable, we can differentiate it to get the first order condition, the envelope condition, and the Euler equation. Differentiating the value function

$$v(H) = \max_{(1-\delta)H \leq H' \leq I/p + (1-\delta)H} \{u(H' - (1-\delta)H, H) + I - p(H' - (1-\delta)H) + \beta v(H')\}$$

we get the FOC

$$u_\phi - p + \beta v'(H') = 0$$

and the ENV

$$v'(H) = -u_\phi(1-\delta) + u_H + p(1-\delta).$$

Joining these two we get the Euler equation

$$\begin{aligned} 0 &= u_\phi(\phi_t, H_t) - p + \\ &\quad + \beta\left[-u_\phi(\phi_{t+1}, H_{t+1})(1-\delta) + u_H(\phi_{t+1}, H_{t+1}) + p(1-\delta)\right], \end{aligned}$$

where $\phi_t = H_{t+1} - (1-\delta)H_t$. This equation, expressed in terms of $F(x, y)$, is

$$0 = F_y(x_t, x_{t+1}) + \beta F_x(x_{t+1}, x_{t+2}).$$

In steady state $\phi^* = \delta H^*$, so the steady state requires that the following equation holds

$$[p - u_\phi(\phi^*, H^*)]\frac{1-\beta(1-\delta)}{\beta} = u_H(\phi^*, H^*).$$

Note that in steady state (and during the time habit is working its way up to H^*) there will be what looks like "overconsumption" of ϕ, i.e. $u_\phi(\phi, H) < p$ in the short run. This is because extra consumption also increases the future utility.

Linearizing the SS equation we get the following coefficients:

- $C(H_t - H^*) = u_{\phi H} - u_{\phi\phi}(1-\delta)$,
- $C(H_{t+1} - H^*) = u_{\phi\phi} + \beta\left[u_{HH} - 2u_{\phi H}(1-\delta) + u_{\phi\phi}(1-\delta)^2\right]$,
- $C(H_{t+2} - H^*) = \beta\left[u_{\phi H} - u_{\phi\phi}(1-\delta)\right]$.

We need to check that

$$F_{xy} = u_{\phi H} - u_{\phi\phi}(1-\delta) \neq 0$$

and

$$\begin{aligned} F_{yy} + (1-\beta)F_{xy} + \beta F_{xx} &= \\ &= \left((1-\beta)(1-\delta) + \delta^2\right)u_{\phi\phi} + (1-\beta+2\beta\delta)\,u_{\phi H} + \beta u_{HH} \\ &\neq 0. \end{aligned}$$

The first is obviously true (it's positive), but the second one is not so obvious. However, it would be quite a coincidence if it were zero, so we can safely assume that it is not.

The characteristic polynomial of this system is

$$\lambda^2 + \lambda\frac{C(H_{t+1} - H^*)}{C(H_{t+2} - H^*)} + \frac{1}{\beta} = 0$$

where $C(H_{t+1} - H^*)$ is the linearization coefficient of $H_{t+1} - H^*$, shown above. Provided that the roots are real, one root, call it λ_1, will be less than one in absolute value, and thus near the steady state the optimal policy function will be

$$H_{t+1} = g(H_t) = H^* + \lambda_1(H_t - H^*).$$

4.2 Two Flavors

When the choice becomes between two flavors, the decision process is more complicated. The following describes how the choice is made and develops the two flavor variant of the model. The second flavor represents mp3s, and its price is the time cost of finding and downloading them. Compared to the time it takes to find and download mp3s, the time that it takes to make mp3s from CDs is small.

In terms of CDs and mp3s, the consumption pattern will be as follows: in the region closer to one's ideal point (i.e. the music that the consumer likes a lot), the consumer will buy CDs. Further from that region, the consumer will download mp3s. He will not attempt to acquire music from very distant regions. Habit will depend on the total amount of music acquired, and will influence the next period's decision. If in a steady state, the price of copying decreases unexpectedly, copying will increase and buying will decrease in the short run. Afterwards, buying will start increasing, but it may never reach the original level. It could but it depends on the relative quality of mp3s and CDs and the strength of the habit effect. Formal development follows.

4.2.1 Utility Fundamentals

Assume there are two flavors of a creative good. For example, in the case of music, the two flavors could be CDs and mp3s. Utility from buying flavor 1 from a set A_1 is

$$u_1(A_1) = q_1 \int_{A_1} v(x)dx,$$

and utility from buying flavor 2 from set A_2 is

$$u_2(A_2) = q_2 \int_{A_2} v(x)dx.$$

If both flavors are consumed from the same set, the utility only depends on the utility the higher quality flavor provides, but both must be paid for. If money enters linearly into the utility function, we can write the music consumer's problem as

$$\max_{A_1, A_2} U(A_1, A_2) = u_1(A_1) + u_2(A_2 \backslash A_1) - p_1(A_1) - p_2(A_2). \tag{3}$$

Proposition 9 *The solution to the consumer's problem defined above is (I) $A_1 = [0, s]$ and $A_2 = [s, \phi_2]$, or (II) $A_1 = [0, \phi_1]$ and $A_2 = \{\emptyset\}$. Here ϕ_1 is the solution to*

$$\begin{aligned} q_1 v(\phi_1) - p_1 &= 0 \textit{ if } \phi_1 > 0 \\ &\leq 0 \textit{ if } \phi_1 = 0, \end{aligned}$$

ϕ_2 is the solution to

$$\begin{aligned} q_2 v(\phi_2) - p_2 &= 0 \textit{ if } \phi_2 > 0 \\ &\leq 0 \textit{ if } \phi_2 = 0, \end{aligned}$$

and s is the solution to

$$\begin{aligned} q_1 v(s) - q_2 v(s) - p_1 + p_2 &= 0 \textit{ if } s > 0 \\ &\leq 0 \textit{ if } s = 0. \end{aligned}$$

If $p_1 < p_2$, then (II) is the solution. Otherwise, (I) is the solution if $s \leq \phi_2$ and (II) is chosen if $\phi_2 < s$. The indirect utility function can be written as

$$\max_{\phi} \begin{cases} u_1(s) + u_2(\phi) - u_2(s) - p_1 s - p_2(\phi - s) & \textit{if } \phi > s \\ u_1(\phi) - p_1\phi & \textit{if } \phi \leq s \end{cases}$$

with s the solution to

$$\begin{aligned} v(s) - \frac{p_1 - p_2}{q_1 - q_2} &= 0 \textit{ if } s > 0 \\ &\leq 0 \textit{ if } s = 0 \\ p_1 - p_2 &< 0 \textit{ if } s = \infty \end{aligned}$$

Discussion: The proposition states that buying always comes before copying and that there may be just copying or just buying. s is the point where buying and copying provides identical marginal utility. $s = 0$ occurs if buying is always worse than copying, and it is possible if $v(0)$ is the right size. If v satisfies the Inada conditions, this will not be the case. If $p_1 < p_2$, there will be no s, as buying is always better (higher quality and cheaper). ϕ_1 and ϕ_2 are the optimal amounts of flavor 1 and flavor 2 if only flavor 1 or only flavor 2 is available. Then the consumer will buy until ϕ_1 or s, whichever is smaller, and copy from s until ϕ_2. Figure 4 is a sketch of all possible two flavor situations.

Proof. We know that if only one flavor exists, the consumption set will be an interval of the form $A_1 = [0, \phi_1]$. If the flavor properties change, it will still have a consumption interval of the form $A_2 = [0, \phi_2]$. Therefore, if two flavors are available, their consumption set will be the interval $A_1 \cup A_2 = [0, \max(\phi_1, \phi_2)]$, with no gaps. There will also be no overlap, i.e. $A_1 \cap A_2 = \{\emptyset\}$.

Now define flavor 1 as the better flavor, i.e. $q_1 > q_2$ and $p_1 > p_2$. Then if a subset of $A_1 \cup A_2$ is in A_1, then any less distant subset will not be in A_2, i.e. if $X \subseteq A_1$, then there is no $Y \subseteq A_2$ s.t. for any $y \in Y$ and $x \in X$ we have $y < x$, for any X and Y that are of positive measure. To see this, say that at some $x > 0$, flavor 1 is better. This implies

$$q_1 v(x) - p_1 > q_2 v(x) - p_2.$$

Then for any $y < x$ we have $v(y) > v(x)$, and so

$$(q_1 - q_2)v(y) > (q_1 - q_2)v(x) > p_1 - p_2$$

i.e. flavor 1 is better at y as well.

It only remains to show what is the point where consumption of flavor one stops and flavor 2 begins. Assume $\phi_1 < \phi_2$, so that the consumption sets are $A_1 = [0, s]$ and $A_2 = [s, \phi_2]$. The FOC of s is

$$\begin{aligned} q_1 v(s) - q_2 v(s) - p_1 + p_2 &= 0 \text{ if } s \in (0, \phi_2) \\ &\leq 0 \text{ if } s = 0 \\ &\geq 0 \text{ if } s = \phi_2. \end{aligned}$$

s will be 0 if $v(0)$ is not large enough to justify buying flavor 1. s will be positive if the solution to

$$v(s) = \frac{p_1 - p_2}{q_1 - q_2}$$

exists for $s \in (0, \phi_2)$. s cannot be ϕ_2, unless $\phi_1 = \phi_2$. It can be shown that if $\phi_1 < \phi_2$, then $s < \phi_1$.

Alternatively, if $\phi_1 > \phi_2$, there will no consumption of flavor 2, as flavor 1 is better at ϕ_1, so at no $y < \phi_1$ will flavor 2 be consumed - and total consumption only reaches to ϕ_1.

If $p_1 < p_2$, there will be no consumption of flavor 2, as flavor 1 is better and cheaper. We can think of s being infinite - at all finite x, flavor 1 is better.

Now to determine what the optimal A_1 and A_2 look like. First look at a restricted maximization: $A_1 = [a_1, b_1]$ and $A_2 = [a_2, b_2]$, with overlap: $0 \le a_1 \le a_2 \le b_1 \le b_2$. Assume also that $v(0) = \infty$, and look at the left hand boundary conditions only. The FOC for a_1,

$$\begin{aligned} -q_1 v(a_1) + p_1 &= 0 \text{ if } a_1 > 0 \\ &\le 0 \text{ if } a_1 = 0 \end{aligned}$$

establishes that $a_1 = 0$. The FOC for a_2,

$$\begin{aligned} p_2 &= 0 \text{ if } a_2 \in (0, b_1) \\ &\le 0 \text{ if } a_2 = 0 \\ &\ge 0 \text{ if } a_2 = b_1 \end{aligned}$$

establishes that $a_2 = b_1$ (of course, overlap is not optimal). The FOC for b_1 is

$$\begin{aligned} q_1 v(b_1) - q_2 v(b_1) - p_1 + p_2 &= 0 \text{ if } b_1 \in (0, b_2) \\ &\le 0 \text{ if } b_1 = 0 \\ &\ge 0 \text{ if } b_1 = b_2 \end{aligned}$$

and it establishes that if $p_1 > p_2$, b_1 is the solution to

$$v(b_1) = \frac{p_1 - p_2}{q_1 - q_2}.$$

If $p_1 < p_2$, then $b_1 = b_2$ for any b_2, i.e. there is no consumption of flavor 2. That's not a surprise, as it is not as good as flavor 1 but more expensive. Finally, the FOC for b_2 depends on whether $b_1 = b_2$ or not. If not, the FOC is just

$$\begin{aligned} q_2 v(b_2) - p_2 &= 0 \text{ if } b_2 > b_1 \\ &\le 0 \text{ if } b_2 = b_1. \end{aligned}$$

If $q_2 v(b_2) - p_2 = 0$ holds for some $b_2 > b_1$, we are done. If $q_2 v(b_1) - p_2 \le 0$ holds, then there is no consumption of flavor 2, as the point where they first become better to consume than flavor 1, it is worse than no consumption.

This case is similar to the above situation with $p_1 < p_2$. Flavor 2 is not consumed, and the problem becomes that of choosing A_1 only, as in the one flavor case. Then we found earlier than $a_1 = 0$ and b_1 solves $q_1 v(b_1) = p_1$.

It is also possible that there is no consumption of flavor 1 and some consumption of flavor 2 - this happens if $v(0)$ is not large enough for flavor 1 and not too small for flavor 2. ■

4.2.2 Consumption over Time

Now that the consumption form is known, we focus on two flavors that are both attractive to consumers, with $q_1 > q_2$ and $p_1 > p_2$. Adding time and habit, as with the single flavor case, we have the following (indirect) consumer's problem in the form of a functional equation:

$$v(H) = \max_{\phi, H'} \left\{ \widehat{U}(\phi, H) + \beta v(H') \right\}$$

where

$$H' = \phi + (1-\delta)H.$$

$\widehat{U}$ is defined by the following equation

$$\widehat{U}(\phi, H) = \begin{cases} q_1 u(s,H) + q_2 u(\phi,H) - q_2 u(s,H) + I - p_1 s - p_2(\phi - s) & \text{if } \phi > s \\ q_1 u(\phi,H) + I - p_1\phi & \text{if } \phi \leq s \end{cases}$$

with s the solution to

$$u_1(s,H) = \frac{p_1 - p_2}{q_1 - q_2}.$$

Focusing on the $\phi > s$ case, here are the first order condition, envelope condition, Euler equation, and steady state characterization. The FOC is

$$q_2 u_1(\phi, H) - p_2 + \beta v'(H') = 0,$$

and the ENV is

$$v'(H) = q_1 u_2(s,H) - q_2 u_1(\phi,H)(1-\delta) + q_2 u_2(\phi,H) - q_2 u_2(s,H) + p_2(1-\delta).$$

Combining them, the Euler equation arises as follows

$$\begin{aligned} 0 \quad = \quad & q_2 u_1(\phi_t, H_t) - p_2 + \\ & +\beta\left[q_1 u_2(s_{t+1}, H_{t+1}) - q_2 u_1(\phi_{t+1}, H_{t+1})(1-\delta) + q_2 u_2(\phi_{t+1}, H_{t+1}) - q_2 u_2(s_{t+1}, H_{t+1}) + p_2(1-\delta)\right] \end{aligned}$$

and in steady state, the following holds

$$0 = \left[q_2 u_1(\phi^*, H^*) - p_2\right](1 - \beta(1-\delta)) + \beta q_2 u_2(\phi^*, H^*) + \beta(q_1 - q_2) u_2(s^*, H^*).$$

In steady state, $\phi^* = \delta H^*$ and $u_1(s^*, H^*) = (p_1 - p_2)/(q_1 - q_2)$. Note that it makes sense to require that

$$\frac{\partial}{\partial H} u_1(\delta H, H) < 0, \tag{4}$$

or more generally that

$$\frac{\partial}{\partial H} u_1(kH, H) < 0, \, \forall k < 1. \tag{5}$$

This way the marginal utility of the last unit consumed in a steady state decreases as the consumption level increases. Thus the long term marginal utility is downward sloping, as expected. I think inequality 5 may not be absolutely necessary, but is needed later for a simple proof that $\partial H \backslash \partial p_2 < 0$.

On to find out how the steady state ϕ, H and s reacts to a change in the price of the lower quality good. An unexpected drop in p_2 would have an immediate effect in the increase of habit H and total information good consumption ϕ. s would drop in the short run, but the rising H would push it up it afterwards.

Differentiating the steady state equation and the equation that defines s with respect to p_2 I get the following

$$\begin{aligned} 0 \quad = \quad & \frac{\partial H}{\partial p_2} q_2 \left[\alpha\delta u_{11}(\delta H, H) + (\alpha + \delta) u_{12}(\delta H, H) + u_{22}(\delta H, H)\right] + \\ & +(q_1 - q_2) u_{22}(s, H) \frac{\partial H}{\partial p_2} - \alpha + (q_1 - q_2) u_{12}(s, H) \frac{\partial s}{\partial p_2} \end{aligned}$$

and

$$u_{11}(s,H)\frac{\partial s}{\partial p_2}+u_{12}(s,H)\frac{\partial H}{\partial p_2}=\frac{-1}{(q_1-q_2)}.$$

Here $\alpha = 1/\beta - (1-\delta)$. Solving for $\partial H \backslash \partial p_2$ one can get $\partial H \backslash \partial p_2 < 0$, since equation 5 assures that

$$\alpha\delta u_{11}(\delta H,H)+(\alpha+\delta)u_{12}(\delta H,H)+u_{22}(\delta H,H)<0$$

and that

$$\alpha+\frac{u_{12}(s,H)}{u_{11}(s,H)}>0.$$

The second claim is true because in steady state $s = kH$ for some $k < \delta$, so $ku_{11}(s,H)+u_{12}(s,H) < 0$ holds. Then $u_{12}(s,H)/u_{11}(s,H) > -k > -\delta$, and therefore

$$\alpha+\frac{u_{12}(s,H)}{u_{11}(s,H)}>\frac{1}{\beta}-1>0.$$

With $\partial H \backslash \partial p_2 < 0$, we can say something about $\partial s \backslash \partial p_2$: it can be positive or negative, and it will be negative if

$$\frac{\partial H}{\partial p_2}<\frac{-1}{(q_1-q_2)u_{12}(s,H)}.$$

This means that the demand for the higher quality flavor is more likely to increase after a decrease of the price of its competitor the larger the quality difference, the larger the effect of the price change on steady state habit, and the larger the effect of habit on marginal utility.

What happens if the price of copying is expected to keep decreasing? The model is not set up to accommodate this, but I expect that the effect of the decreasing price will be even stronger, and that the demand for originals will rebound less strongly.

4.3 N Flavors

This section sketches an information good with many flavors. Assume there are N flavors of the information good, each with its quality level and price. Flavor i, with quality level q_i, results in a $q_i u(s,H)$, as for the two flavor case.

If the consumer has a choice about the flavor of the good he consumes, he will consider his net marginal utility of consuming any one flavor at a specific point of the information good space, and he will choose the flavor that provides him with the most utility. At a point s, the net marginal utility of flavor i is $q_i u(s,H) - p_i$. If there are two flavors, i and j, with $q_i > q_j$, both will be considered only if $p_i > p_j$. If the lower quality flavor is more expensive than the higher quality flavor, it cannot provide a higher net marginal utility. If $p_i > p_j$, the two flavors will provide the same amount of net marginal utility at a point s where

$$u_1(s,H)=\frac{p_i-p_j}{q_i-q_j}.$$

If there are N flavors available to the consumer, ordered by decreasing quality, the consumer will reject any flavor for which there exists a better and cheaper flavor. Thus only flavors like these will be considered:

$$\begin{aligned} q_1 &> q_2 > q_2 > \ldots > q_N \\ p_1 &> p_2 > p_1 > \ldots > p_N. \end{aligned}$$

Even then, some flavors may not make it onto the consumers to buy list: there may be flavors for which there exists no point z s.t.

$$q_i u_1(z,H) - p_i > \max_{j \neq i} \{q_j u_1(z,H) - p_j\}$$

After discarding such flavors, assume that there are N flavors left. The $N-1$ switching points

$$s_1 < s_2 < \ldots < s_{N-1}$$

are those where the consumer switches flavors, from 1 to 2, 2 to 3, and so on until the $N-1$ to N switch. Then the consumer's problem becomes where to stop the consumption. The functional equation for this problem becomes

$$v(H) = \max_{\phi, H'} \left\{ \widehat{U}(\phi, H) + \beta v(H') \right\}$$

where

$$H' = \phi + (1-\delta)H.$$

$\widehat{U}$ is defined by the following equation

$$\begin{aligned} \widehat{U}(\phi,H) &= q_1 u(s_1,H) + \sum_{i=2}^{K-1} [q_i u(s_i,H) - q_i u(s_{i-1},H)] + q_K u(\phi,H) - q_{K-1} u(s_{K-1},H) \\ &\quad + I - p_1 s_1 - \sum_{i=2}^{K-1} p_i(s_i - s_{i-1}) - p_K(\phi - s_{K-1}). \end{aligned}$$

Here K is the last flavor to be consumed. If $\phi > s_{N-1}$, then $K = N$. Otherwise, K is defined by $s_K < \phi < s_{K+1}$. If $\phi < s_1$, $\widehat{U}(\phi,H) = q_1 u(\phi,H) + I - p\phi$.

5 Examples

This section uses explicit utility functions to determine the general requirements for an increase in the consumption of the better flavor when the price of the lesser flavor decreases. The parallel with the world of copying is immediate: the better flavor is are CDs, with their higher quality. The lesser flavor are mp3's, with their lower quality and lower price (the price is lower for those that choose to consume mp3's). The examples follow.

5.1 Cobb Douglas and CES Utility

I compute the model for one and two flavors using Cobb Douglas and constant elasticity of substitution (CES) utility functions. The time preference parameter is set to $\beta = 0.95$, and habit depreciates at a rate $\delta = 0.1$. The Cobb Douglas utility is inside a constant relative risk aversion (CRRA) function

$$u(\phi,H) = \frac{(\phi^\alpha H^\gamma)^{1-\rho}}{1-\rho},$$

with $\alpha + \gamma < 1$ and $1 - \rho > 0$ necessary for $u_{12} < 0$. The CES function is

$$u(\phi,H) = [\alpha_1 \phi^\rho + \alpha_2 H^\rho]^{m/\rho},$$

with $\alpha_1 + \alpha_2 < 1$, $\rho \in (0,1)$ and $m \in (\rho,1)$.

Approximate $v(H)$ and $g(H)$ were calculated with shape-preserving quadratic splines (Schumaker (1983), see Judd p#231) with 27 equally spaced node points in the interval $[0.1H^*, 2H^*]$. H^* is found from Euler equation in SS. Iteration are performed on the value function and policy function.

Figure 5 contains graphs of the computed value, policy and other functions for a specific parametrization of the two flavor Cobb Douglas case. Figure 6 shows a path to steady state for the same parametrization. Note that when the habit is very low, copying is very high and decreasing with the habit increasing. The future high habit has a strong effect on current consumption.

Figures 7 and 8 show how the steady state of habit and originals and copy consumption depends on the cost of copying (still Cobb Douglas). When the cost is high, there is no copying, but as the cost drops below a certain threshold, copying starts. In Figure 7, demand for originals never recovers, but in Figure 8, it starts increasing once the cost of copying is very small.

Figures 9 and 10 are examples of the paths that habit and consumption take after a price change. The price changes are taken from Figure 8. In Figure 9, demand for originals (s) ends up higher than it started, while in Figure 10 it does not. Notice that in Figure 9, although demand for originals falls immediately after the price change, it increases above the previous steady state level in the second period after the price change. That seems to be too soon for a realistic modelling of the demand for originals.

To see that $\partial s/\partial p_2 < 0$ is indeed possible throughout the entire range of p_2, Figures 11 and 12 provide two examples. Figure 11 is for Cobb Douglass and Figure 12 for CES utility. To achieve this, the parametrization must be extreme, and it is thus unlikely that it will happen in the real world.

5.2 Quadratic Utility, $\delta = 1$

The marginal utility in Johnson's (1985) buy decision rule is linear, which implies quadratic utility. Assume U is of the form

$$U(\phi, z, H) = -\frac{1}{2}q(\mu(H) - \phi)^2 + \frac{1}{2}qK\mu(H)^2 + z. \tag{6}$$

K is a constant, q is quality, and μ is an increasing function of habit. The budget constraint is $p\phi + z = I$, and the law of motion for habit is $H' = \phi$.

This utility function is increasing and concave in ϕ when H is taken as given as well as when $H = \phi$ and μ is well behaved (not increasing too fast). This utility may result in no consumption, as the marginal utility does not increase without bound for small consumptions. The following analysis is concerned with cases when there is consumption.

The utility of consuming no music can increase or decrease with habit, as determined by K. $K = 1$ sets $u(0, z, H) = z$ regardless of H. It makes sense that $K < 1$, so that higher habit results in more unhappiness if consumers are forced to consume little of their beloved music. I will however, not consider this case here.

The dynamic problem is

$$\max_{\{\phi_t, z_t\}} \sum_{t=0}^{\infty} \beta^t U(\phi_t, z_t, H_t)$$

such that

$$H_{t+1} = \phi_t.$$

The FOC in steady state is

$$-q(\mu - \phi)(\beta\mu' - 1) + \beta q\mu\mu' = p.$$

A linear μ,

$$\mu(\phi) = \mu(0) + \mu'\phi,$$

with scalars $\mu(0)$ and μ', allows for an explicit solution for the steady state ϕ, and it is

$$\phi^{SS} = \frac{\mu(0) - p/q}{1 - \mu'(1+\beta)}.$$

Thus $\phi^{SS} > 0$ if $\mu' < 1/(1+\beta)$ and $\mu(0) > p/q$. In that case we also have $\partial \phi^{SS}/\partial p < 0$. Therefore, $\mu' < 1/(1+\beta)$ is required for a well behaved solution.

With two flavors available, this problem becomes

$$\max_{\{\phi_t, s_t, z_t\}} \sum_{t=0}^{\infty} \beta^t \widehat{U}(\phi_t, s_t, z_t, H_t)$$

where

$$\widehat{U}(\phi, s, z, H) = -\frac{1}{2} q_1 (\mu - s)^2 - \frac{1}{2} q_2 [(\mu - \phi)^2 - (\mu - s)^2] + \frac{1}{2} q_1 \mu^2 + z$$

and the law of motion for H is $H_{t+1} = \phi_t$. The budget constraint is $p_1 s + p_2(\phi - s) + z = I$. The FOC's are

$$(\mu - s_t)(q_1 - q_2) = p_1 - p_2$$

and

$$q_2(\mu - \phi_t) - p_2 + \beta \left[-q_1(\mu - s_{t+1})\mu' - q_2[(\mu - \phi_{t+1}) - (\mu - s_{t+1})]\mu'] + q_1 \mu \mu' \right] = 0.$$

In steady state, $\phi_t = \phi_{t+1} = \phi^{SS}$ and $s_t = s_{t+1} = s^{SS}$. Combining the two equations, assuming a linear μ, ($\mu(\phi) = \mu(0) + \mu'\phi$) and differentiating with respect to p_2, we get

$$\frac{\partial \phi}{\partial p_2} = \frac{1 - \beta\mu'}{q_2(\mu'(1+\beta) - 1) + \beta \mu'^2 (q_1 - q_2)}.$$

This is negative when

$$\mu' < \frac{1}{2\beta(q_1 - q_2)} \left[\sqrt{q_2^2(1-\beta)^2 + 4\beta q_1 q_2} - q_2(1+\beta) \right].$$

Note that as $\beta \to 1$ this upper bound tends to

$$\frac{\sqrt{q_C}}{\sqrt{q_B} + \sqrt{q_C}} < \frac{1}{2}.$$

We also want to know the sign of $\partial s/\partial p_2$. From the first FOC we know that (with linear μ)

$$\frac{\partial s}{\partial p_2} = \frac{1}{q_1 - q_2} + \mu' \frac{\partial \phi}{\partial p_2}.$$

Combining these two derivatives, and some hairy algebra, it turns out that $\partial s/\partial p_2 < 0$ if

$$\mu' > \frac{1}{2q_1(1-\beta)} \left[\sqrt{(q_1 - \beta q_2)^2 + 4 q_1 q_2} - (q_1 + \beta q_2) \right].$$

This lower bound will tend to

$$\frac{q_C}{q_B + q_C} < \frac{\sqrt{q_C}}{\sqrt{q_B} + \sqrt{q_C}}$$

as $\beta \to 1$.

With this utility specification, the cross derivative, $u_{12}/q = \mu'$, cannot be too large, or long run demand functions will look perverse. But if it is not too small, then a decrease of the price of copying will initially decrease the demand for originals, but it will eventually increase it to a level larger than what it initially was. As $\beta \to 1$, the region in which this will happen converges to

$$\mu' \in \left[\frac{q_C}{q_B + q_C}, \frac{\sqrt{q_C}}{\sqrt{q_B} + \sqrt{q_C}} \right]$$

5.3 Quadratic Utility, $\delta = 1,\ \beta = 0$

An even simpler habit forming model is one in which the consumer is not aware of the effect his current consumption will have on the future habit and therefore the future utility. This will be true if the consumer is myopic, but it does not mean that he does not care about the future, as one may presume from $\beta = 0$, but that he does not see the law of motion of habit in effect. This model is extensively analyzed in Grgeta (2002).

Another possibility for this myopia is that habit is an economy wide variable. If all people have the same habit, then no one person can influence the future habit by his own consumption only. Perhaps habit can be local, so that similar people have similar habits, but any one person is still one of many with such habit.

Here the utility function is as defined in equation 6, with $K = 1$, and $H_{t+1} = \phi_t$. This is basically Johnson's (1985) household copying model with varying quality and habit. The FOC for one flavor only is

$$q(\mu - \phi) = p$$

which implies that

$$\phi = \mu - p/q.$$

In the steady state we will have

$$\phi^{SS} = \mu(\phi^{SS}) - p/q.$$

With a linear μ ($\mu(\phi) = \mu(0) + \mu'\phi$)

$$\phi^{SS} = \frac{\mu(0) - p/q}{1 - \mu'}.$$

This problem is well defined (and has a nice solution) if $\mu' < 1$ and $\mu(0) > p/q$. Note that a person with $\mu(0) \leq p/q$ will not consume any of this good.

With two available flavors, the problem becomes similar to the one above, but with $\beta = 1$. The FOC's are

$$\begin{aligned} (q_1 - q_2)(\mu - s) &= p_1 - p_2 \\ q_2(\mu - \phi) &= p_2. \end{aligned}$$

These characterize the optimal s and ϕ, and determine a steady state in which the following equations hold:

$$\begin{aligned} (q_1 - q_1)(\mu(\phi^{SS}) - s^{SS}) &= p_1 - p_2 \\ q_1(\mu(\phi^{SS}) - \phi^{SS}) &= p_2. \end{aligned}$$

See Grgeta (2002) for details. The interesting demand function derivatives are then

$$\begin{aligned} \frac{\partial \phi^{SS}}{\partial p_2} &= \frac{-1}{q_2(1 - \mu')} \\ \frac{\partial s^{SS}}{\partial p_2} &= \frac{1}{q_1 - q_2} - \frac{\mu'}{q_2(1 - \mu')}. \end{aligned}$$

If μ' is relatively large or the quality difference is relatively large, then a decrease of the price of the lesser flavor will increase the steady state consumption of the better flavor. Formally, if $\mu' > q_2/q_1$, then $\partial s^{SS}/\partial p_2 < 0$, that is, the flavors are *complements in steady state*, but *instantaneous substitutes*: $\partial \phi^{SS}/\partial p_2|_{\mu=\overline{\mu}} = 1/(q_1 - q_2))$. Note that the consumption of the lesser flavor will always increase with the decrease of its price: $\partial(\phi^{SS} - s^{SS})/\partial p_2 < 0$.

In the language of CDs and mp3s, if the relative quality of mp3s is low and the habit effect is strong ($q_{mp3}/q_{CD} < \mu' < 1$), then a decrease of the cost of mp3s will initially decrease the demand for CDs, but eventually it will rise above its original level. Throughout the adjustment time, copying is increasing - thus the observation that an increase in copying follows a decrease in copying costs does not tell us the long run effect on originals.

Figure 13 shows the copy/buy decision of consumers at any point of the line for some distribution of a and ω. ($a = \mu(0)$, ω is the cost of copying, b is $\mu' < q_{mp3}/q_{CD}$ here, p is the price of CDs and ψ is the distance to the music). Note the difference from Figure 1: a group of people that copied in Figure 1 now buys CDs. If instead $\mu' > q_{mp3}/q_{CD}$, fewer people would buy, but still more than in Johnson (1985).

6 Introducing the Third Flavor

This section will introduce a third flavor. This flavor will be offered by the copyright owners, so its price and quality will be adjustable to maximize some measure of profit.

[TO BE COMPLETED SOON]

6.1 Theory

[THIS SUBSECTION WILL CONTAIN COMPUTATIONS FOR THE INTRODUCTION OF A THIRD FLAVOR]

6.2 Practice

[THIS SUBSECTION WILL DESCRIBE THE ATTEMPTS OF THE RECORD COMPANIES AT INTRODUCING THE THIRD FLAVOR]

7 Conclusion

[ROUGH]

This paper models demand for originals in the presence of copies. It differs from other research in that area because it does not force copies and originals to be of the same quality, introduces a habit variable, and allows the buying and copying decision to occur over time.

The main question of this paper is: can an increase in copying (from a decrease in copying costs, say) increase the demand for originals. The model shows that increased copying can increase demand, but it is unlikely this will happen. In addition, the paper asks whether producers of originals can increase their revenues in the face of growing copying. [THIS PART INCOMPLETE]

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Figure 1

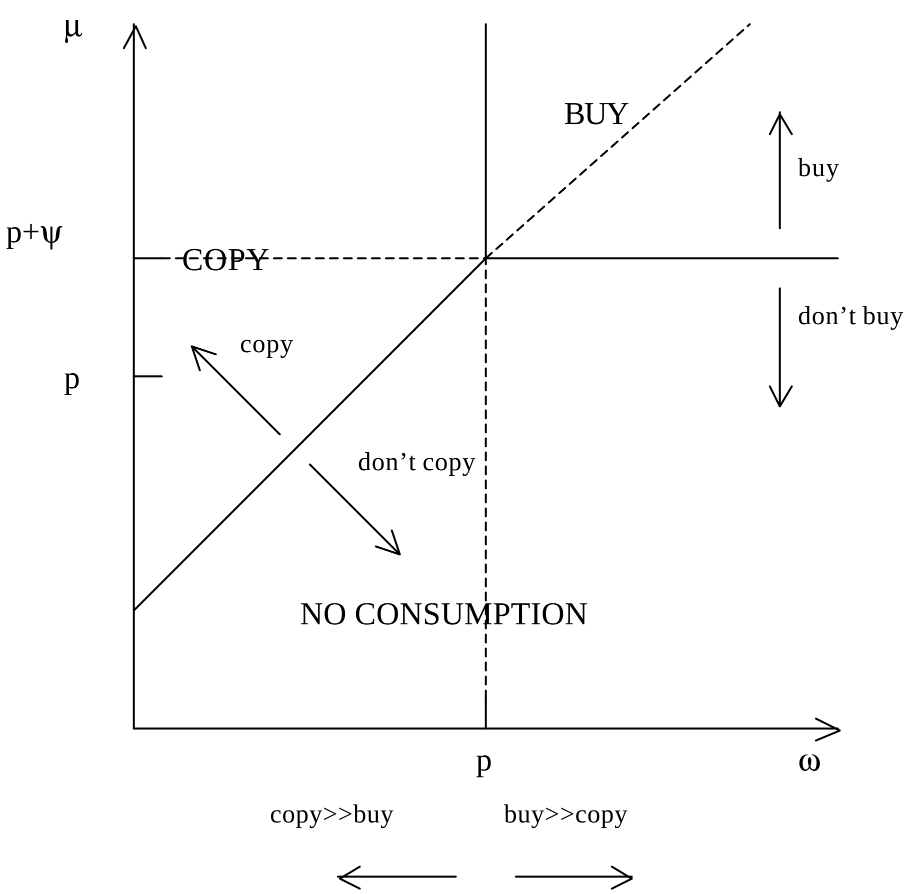
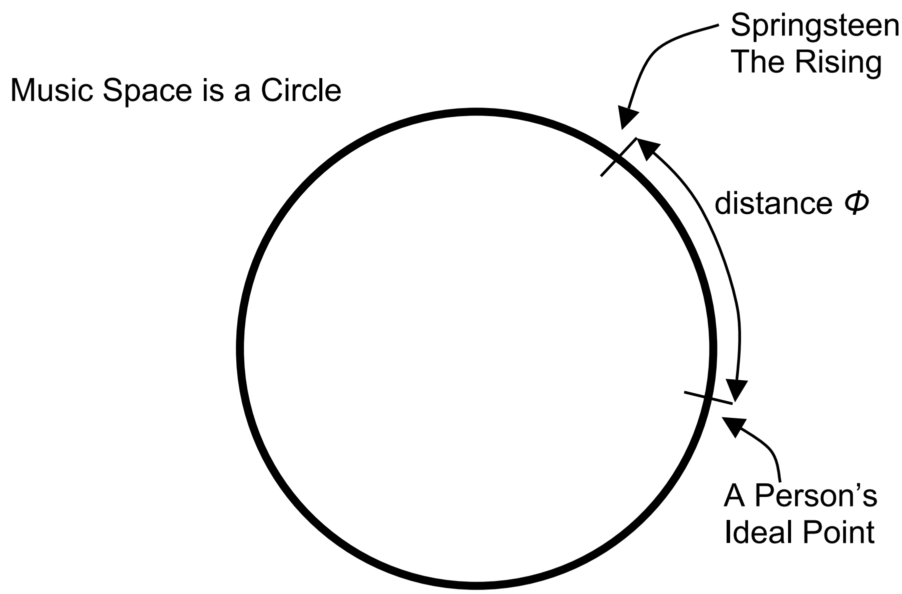
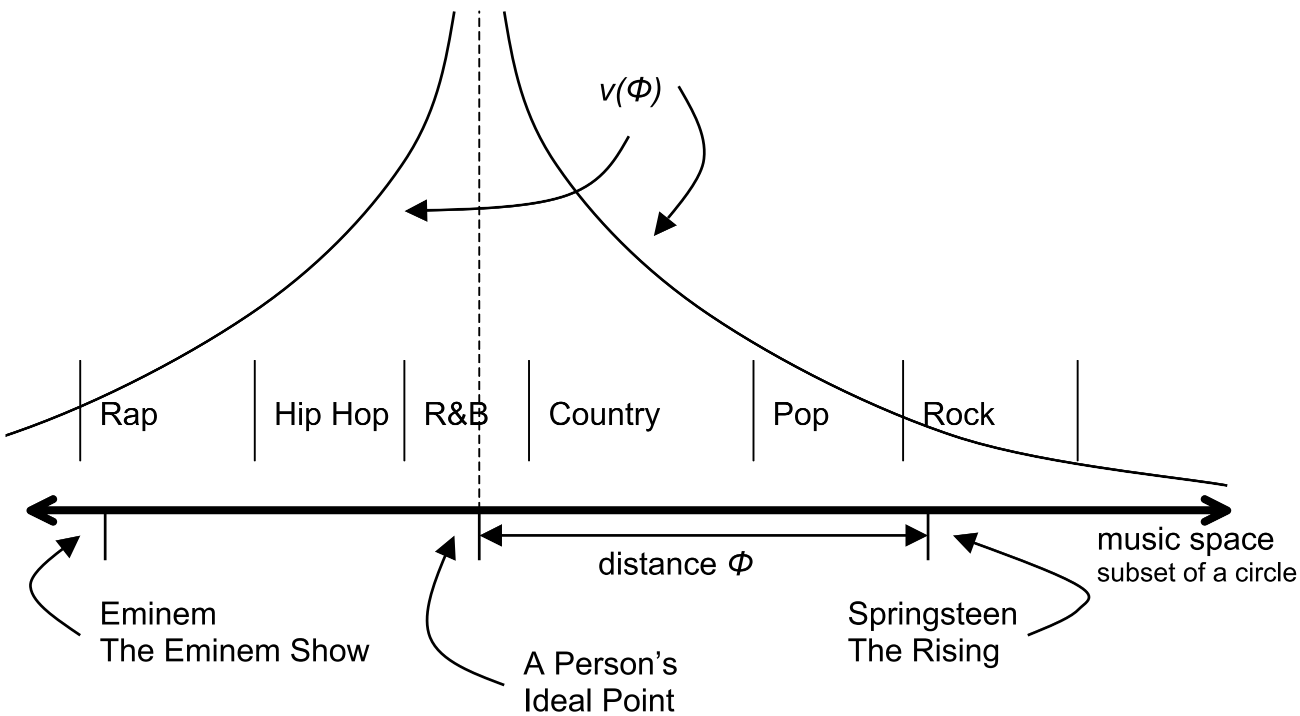

μ
p+ψ
p
BUY
buy
COPY
copy
don’t buy
don’t copy
NO CONSUMPTION
p
ω
copy>>buy
buy>>copy

Figure 2



Music Space is a Circle
Springsteen
The Rising
distance Φ
A Person's
Ideal Point



v(Φ)
Rap
Hip Hop
R&B
Country
Pop
Rock
music space
subset of a circle
distance Φ
Eminem
The Eminem Show
A Person's
Ideal Point
Springsteen
The Rising

Figure 3

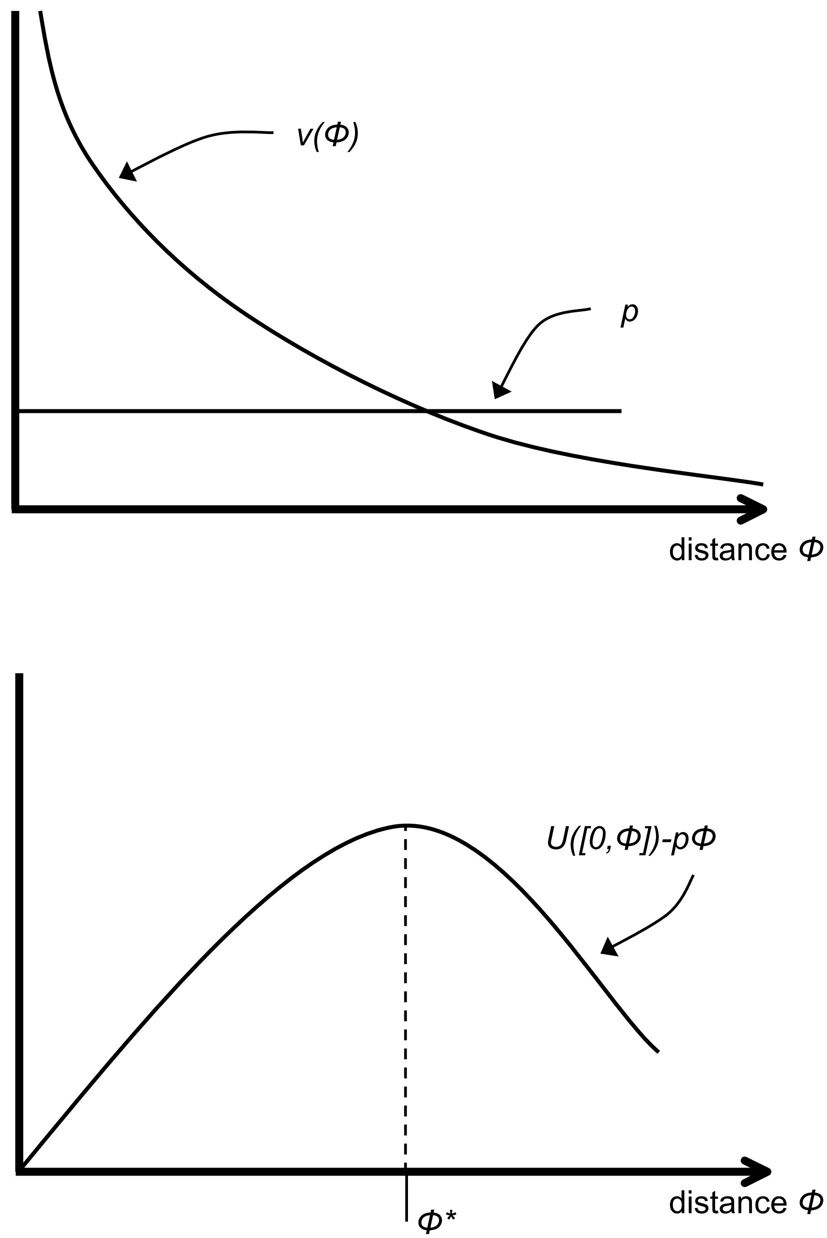
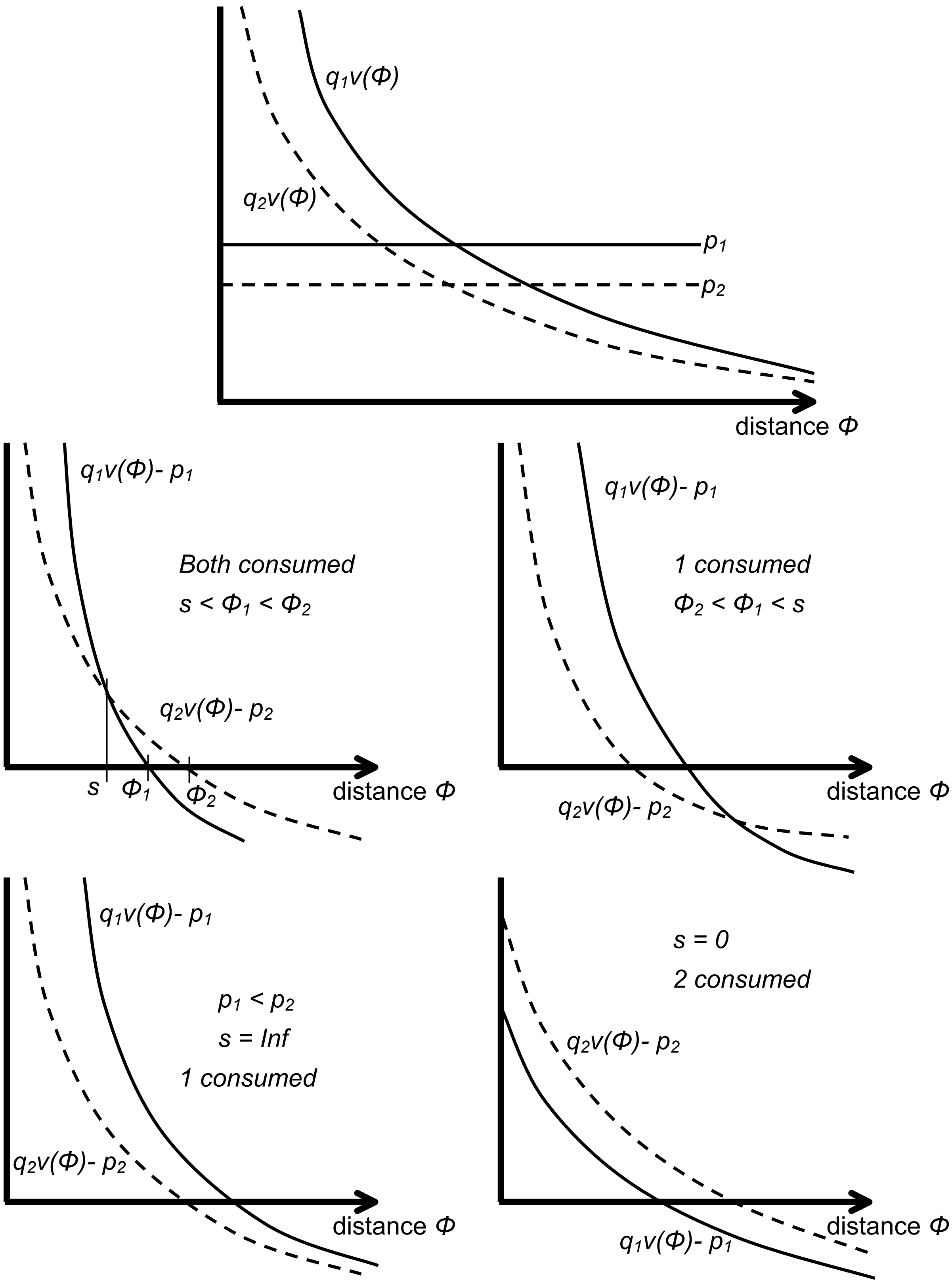
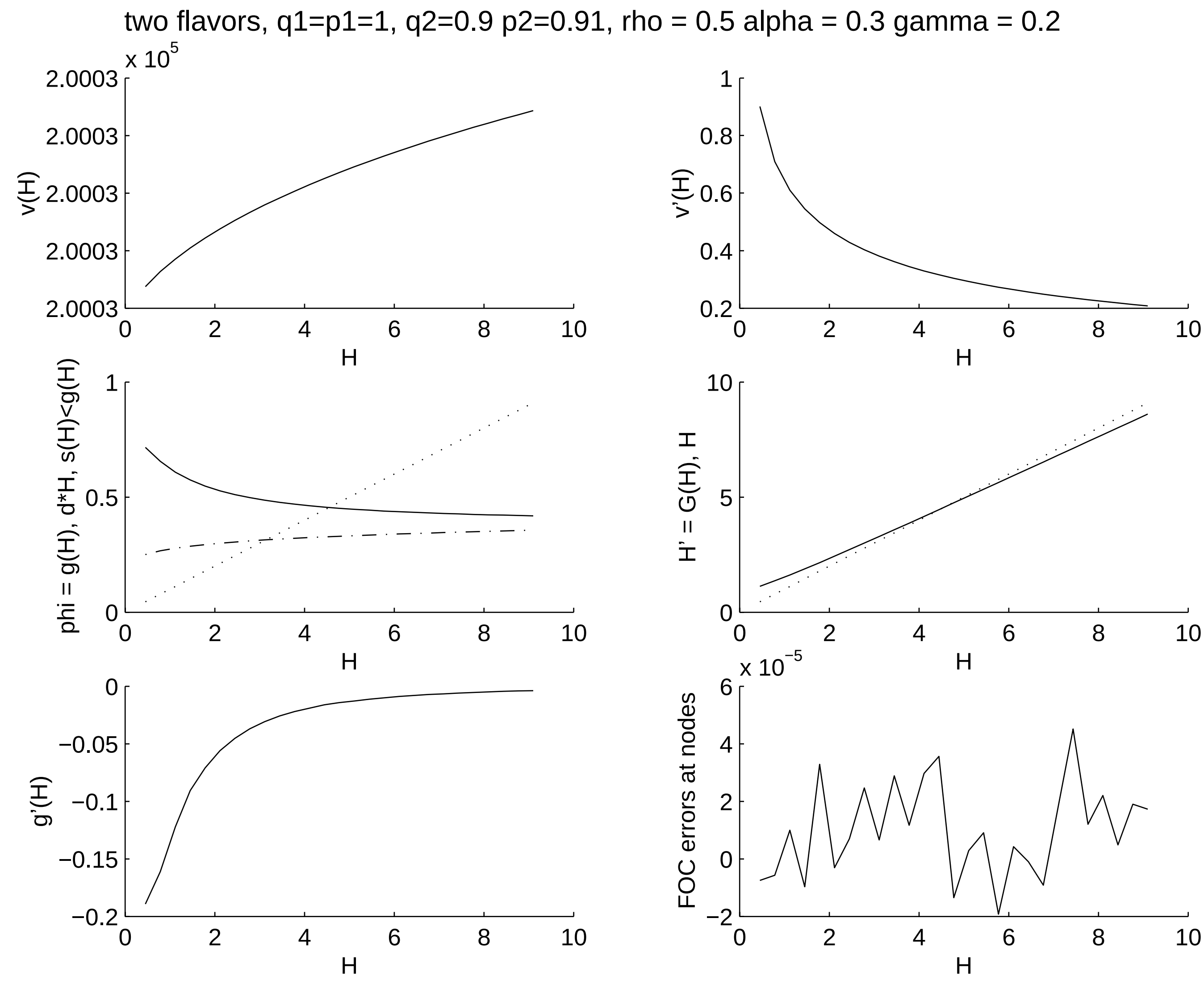


Figure 4



q1v(Φ)
q2v(Φ)
p1
p2
distance Φ
q1v(Φ)- p1
Both consumed
s < Φ1 < Φ2
q2v(Φ)- p2
s
Φ1
Φ2
distance Φ
q1v(Φ)- p1
1 consumed
Φ2 < Φ1 < s
distance Φ
q2v(Φ)- p2
q1v(Φ)- p1
p1 < p2
s = Inf
1 consumed
q2v(Φ)- p2
distance Φ
s = 0
2 consumed
q2v(Φ)- p2
distance Φ
q1v(Φ)- p1

Figure 5



two flavors, q1=p1=1, q2=0.9 p2=0.91, rho = 0.5 alpha = 0.3 gamma = 0.2
x 10^5
2.0003
2.0003
2.0003
2.0003
2.0003
v(H)
0 2 4 6 8 10
H
1
0.8
0.6
0.4
0.2
v'(H)
0 2 4 6 8 10
H
1
0.5
0
phi = g(H), d*H, s(H)<g(H)
0 2 4 6 8 10
H
10
5
0
H' = G(H), H
0 2 4 6 8 10
H
0
−0.05
−0.1
−0.15
−0.2
g'(H)
0 2 4 6 8 10
H
x 10^−5
6
4
2
0
−2
FOC errors at nodes
0 2 4 6 8 10
H

Figure 6

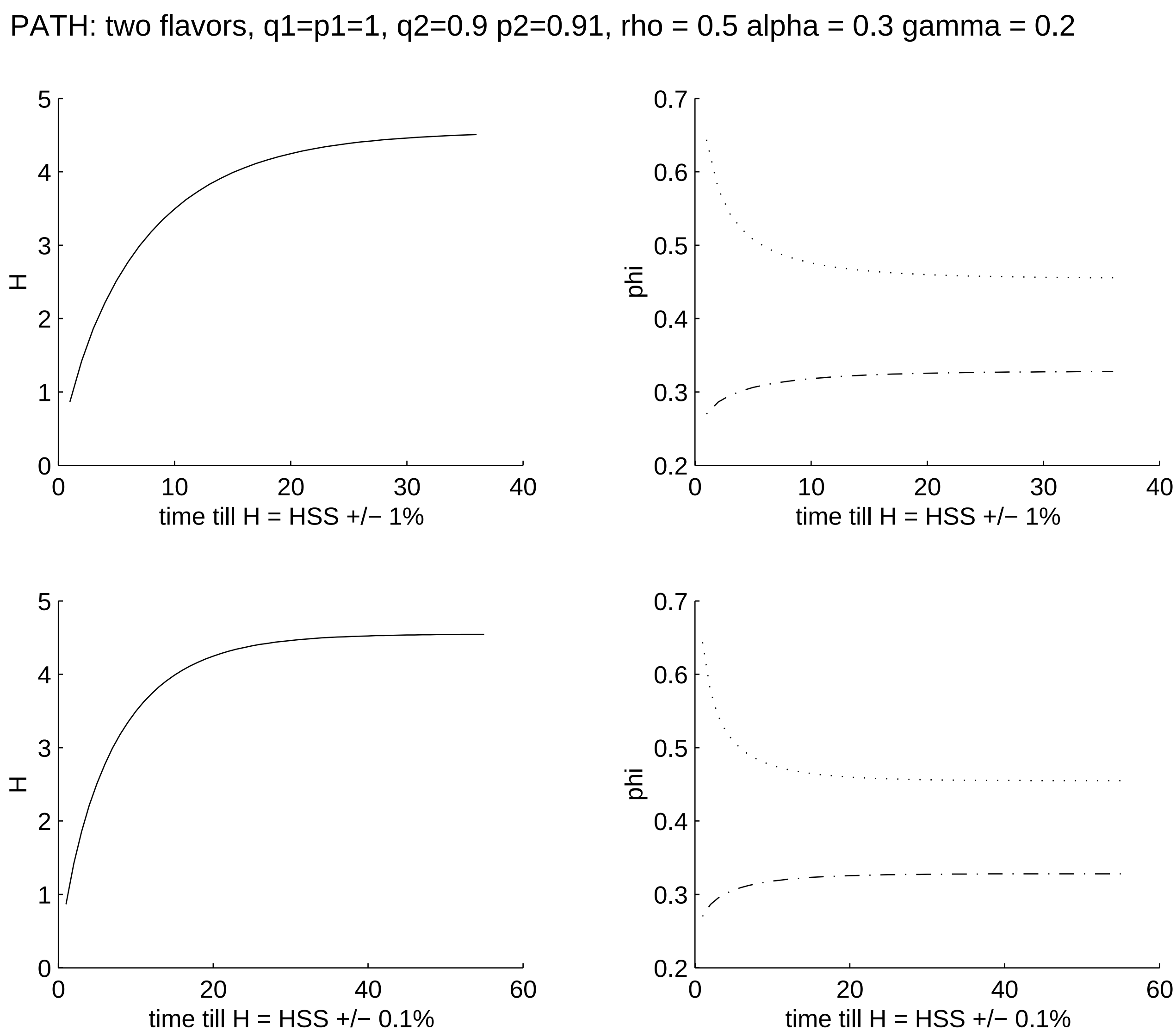

PATH: two flavors, q1=p1=1, q2=0.9 p2=0.91, rho = 0.5 alpha = 0.3 gamma = 0.2
H
time till H = HSS +/− 1%
phi
time till H = HSS +/− 1%
H
time till H = HSS +/− 0.1%
phi
time till H = HSS +/− 0.1%

Figure 7

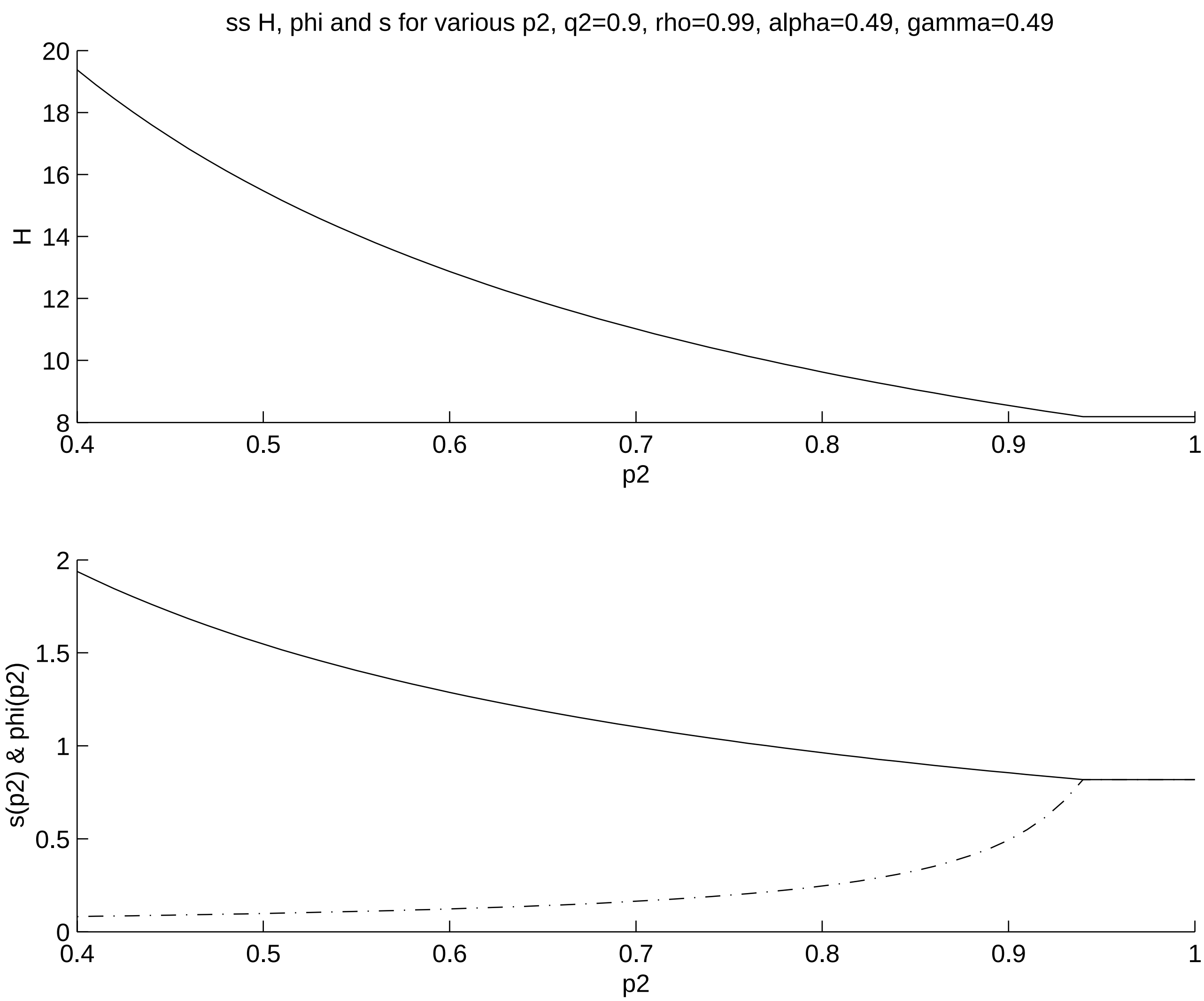
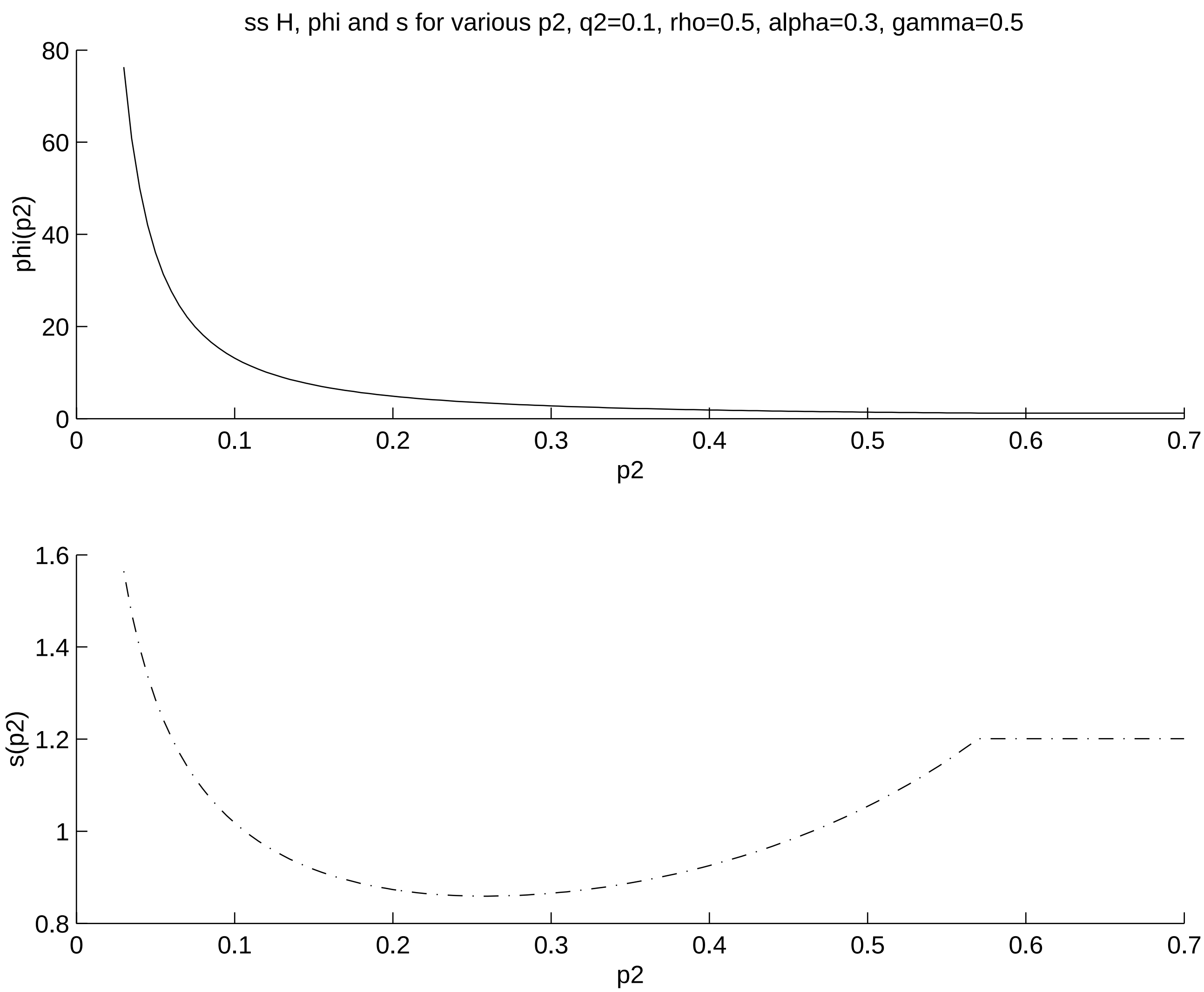
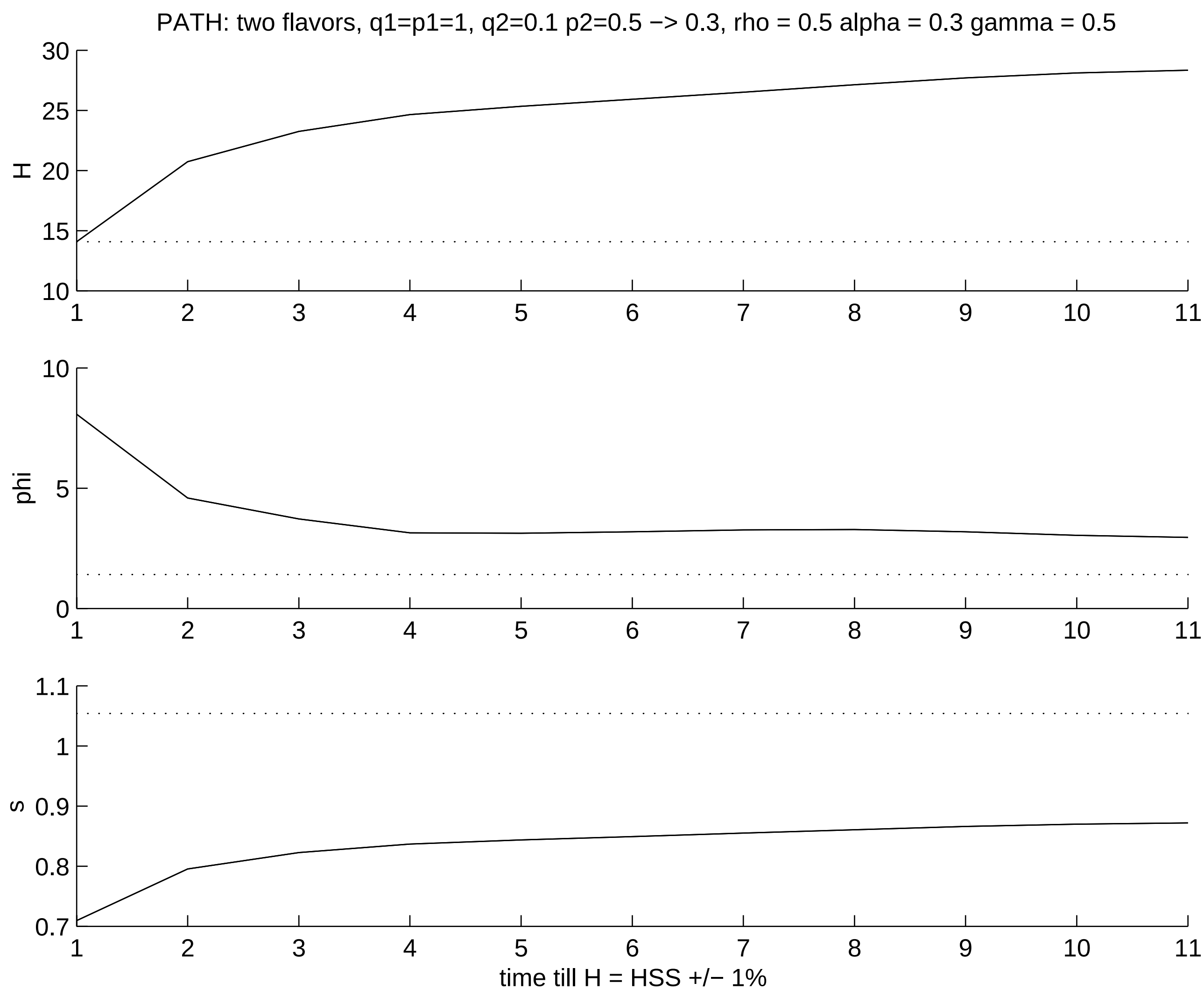

ss H, phi and s for various p2, q2=0.9, rho=0.99, alpha=0.49, gamma=0.49
H
20
18
16
14
12
10
8
0.4
0.5
0.6
0.7
0.8
0.9
1
p2
s(p2) & phi(p2)
2
1.5
1
0.5
0
0.4
0.5
0.6
0.7
0.8
0.9
1
p2

Figure 8



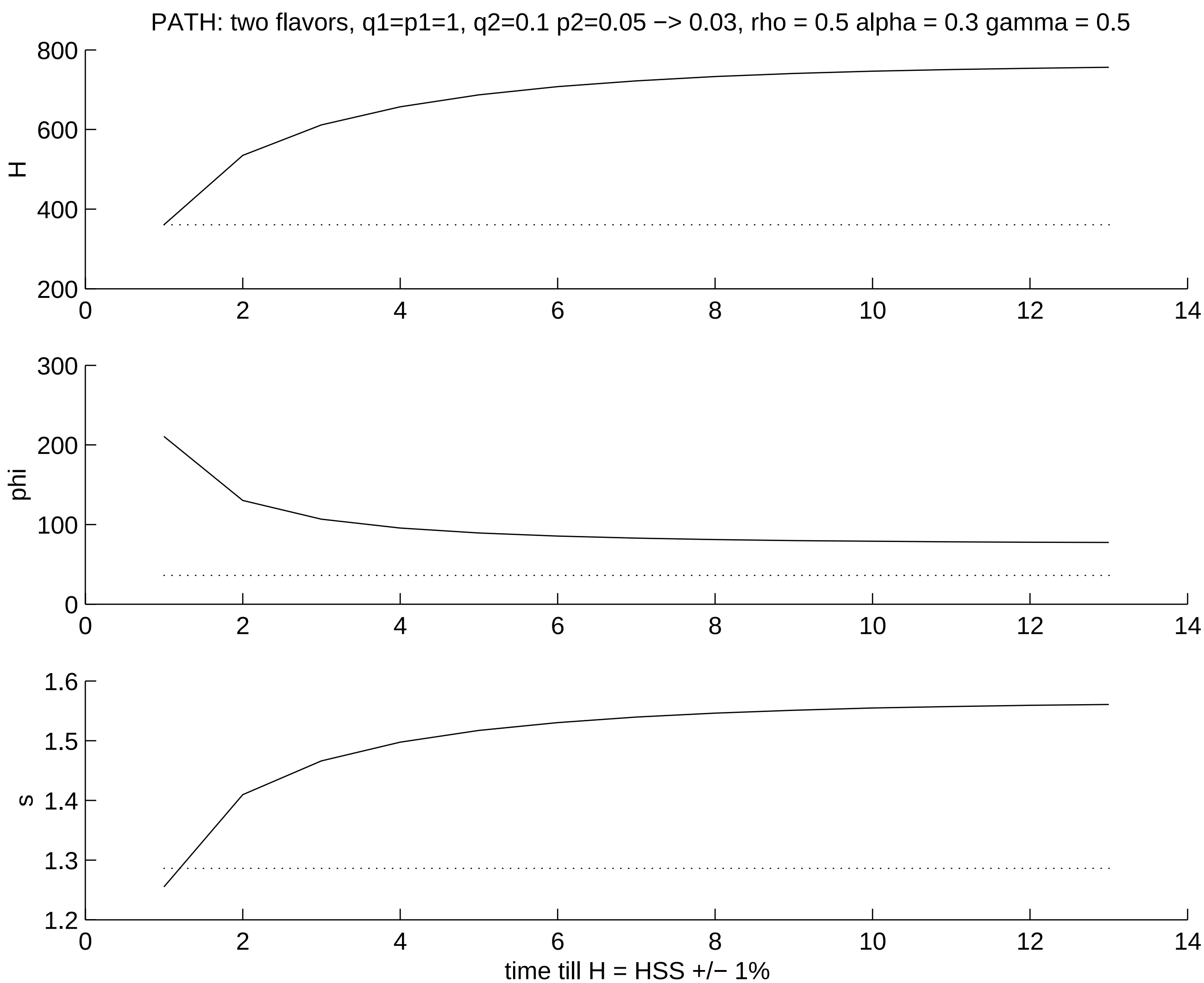
ss H, phi and s for various p2, q2=0.1, rho=0.5, alpha=0.3, gamma=0.5
phi(p2)
80
60
40
20
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
p2
s(p2)
1.6
1.4
1.2
1
0.8
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
p2

Figure 9



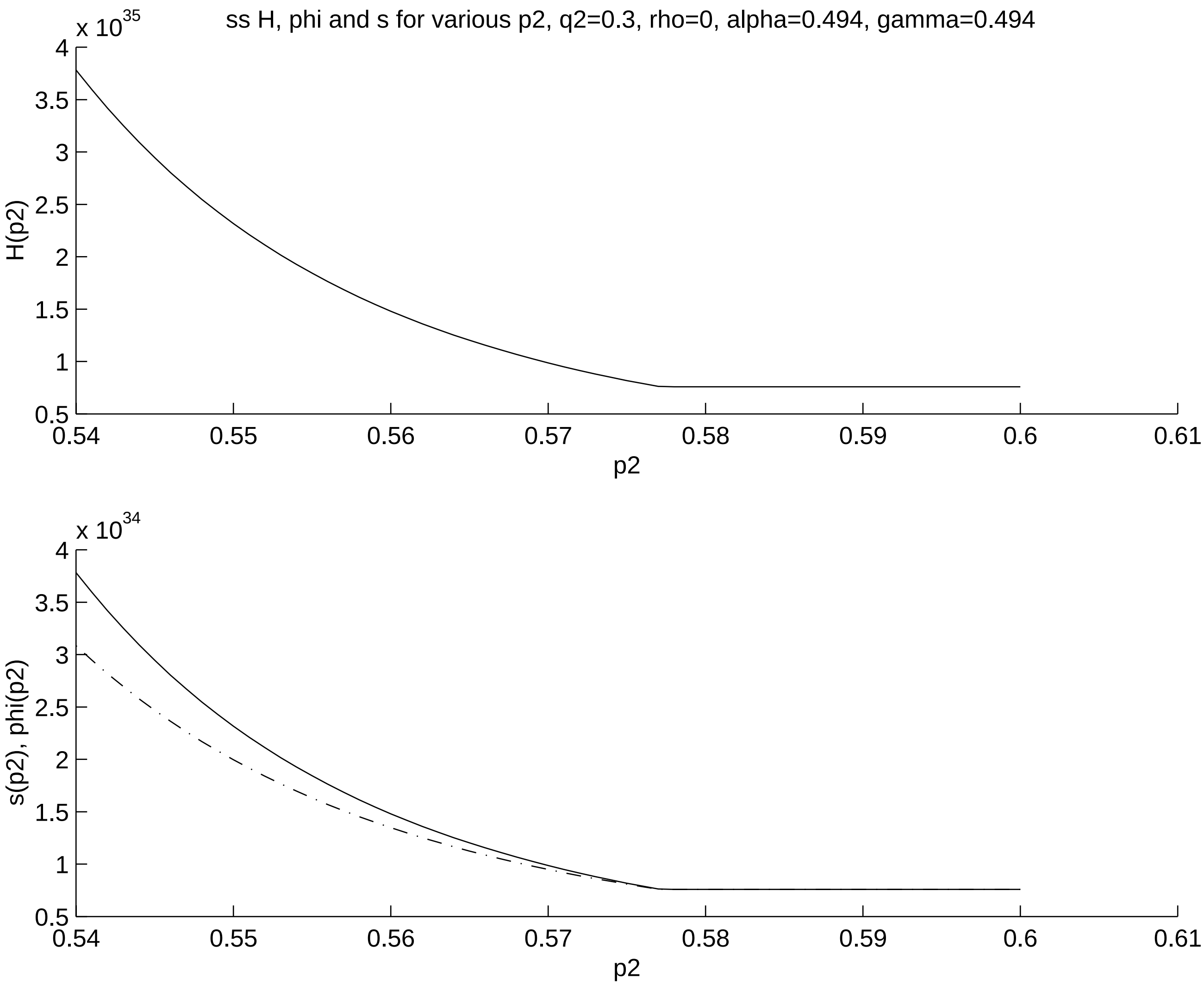
PATH: two flavors, q1=p1=1, q2=0.1 p2=0.5 −> 0.3, rho = 0.5 alpha = 0.3 gamma = 0.5
H
30
25
20
15
10
1 2 3 4 5 6 7 8 9 10 11
phi
10
5
0
1 2 3 4 5 6 7 8 9 10 11
s
1.1
1
0.9
0.8
0.7
1 2 3 4 5 6 7 8 9 10 11
time till H = HSS +/− 1%

Figure 10



PATH: two flavors, q1=p1=1, q2=0.1 p2=0.05 −> 0.03, rho = 0.5 alpha = 0.3 gamma = 0.5
H
800
600
400
200
0
2
4
6
8
10
12
14
phi
300
200
100
0
s
1.6
1.5
1.4
1.3
1.2
time till H = HSS +/− 1%

Figure 11



ss H, phi and s for various p2, q2=0.3, rho=0, alpha=0.494, gamma=0.494
x 10^35
H(p2)
4
3.5
3
2.5
2
1.5
1
0.5
0.54
0.55
0.56
0.57
0.58
0.59
0.6
0.61
p2
x 10^34
s(p2), phi(p2)
4
3.5
3
2.5
2
1.5
1
0.5
0.54
0.55
0.56
0.57
0.58
0.59
0.6
0.61
p2

Figure 12

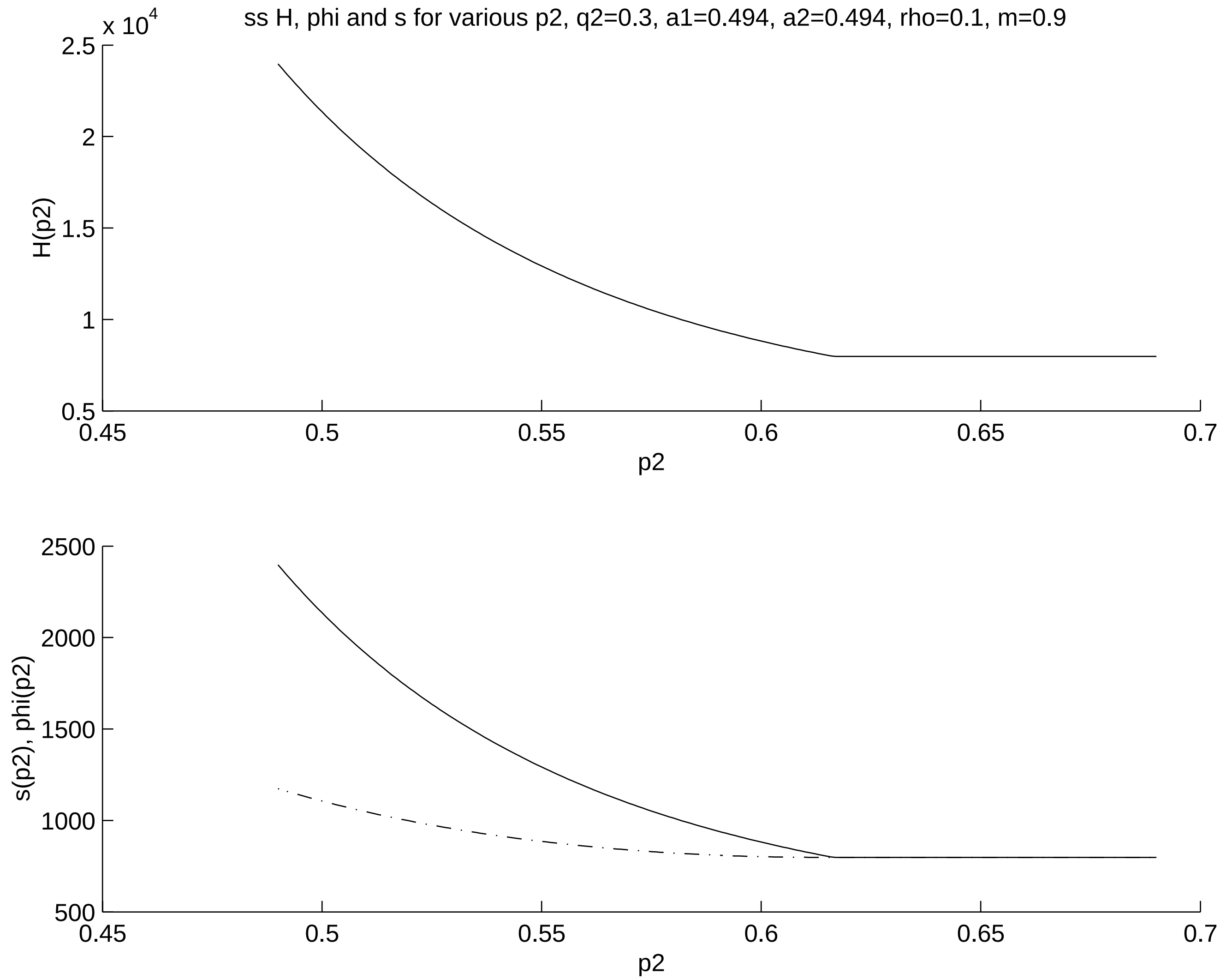

ss H, phi and s for various p2, q2=0.3, a1=0.494, a2=0.494, rho=0.1, m=0.9
x 10^4
H(p2)
2.5
2
1.5
1
0.5
0.45
0.5
0.55
0.6
0.65
0.7
p2
s(p2), phi(p2)
2500
2000
1500
1000
500
0.45
0.5
0.55
0.6
0.65
0.7
p2

Figure 13