

# Pricing information goods in the presence of copying

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## Abstract

The effects of piracy on the pricing behavior of producers of information goods is studied within a unified model à la Mussa-Rosen (1978). When the copying technology involves a marginal cost and no fixed cost, producers act independently. In this simple framework, we highlight the trade-off between *ex ante* and *ex post* efficiency considerations (how to provide the right incentives to create whilst limiting monopoly distortions?). When the copying technology involves a fixed cost and no marginal cost, pricing decisions are interdependent. We investigate the strategic pricing game by focussing on some significant symmetric Nash equilibria.

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# 1 Introduction

Information can be defined very broadly as anything that can be digitized (i.e., encoded as a stream of bits), such as text, images, voice, data, audio and video (see Varian, 1998). Information is exchanged under a wide range of formats or packages (which are not necessarily digital). These formats are generically called *information goods*. Books, movies, music, magazines, databases, telephone conversations, stock quotes, web pages, news, etc all fall into this category.

Most information goods are expensive to produce but cheap to reproduce. This combination of high fixed costs and low (often negligible) marginal costs implies that information goods are inherently *nonrival*.<sup>1</sup> Moreover, because reproduction costs are also potentially very low *for anybody else* than the creator of the good, information goods might be *nonexcludable*, in the sense that one person cannot exclude another person from consuming the good in question.

The degree of excludability of an information good (and hence the creator's ability to appropriate the revenues from the production of the good) can be enhanced by legal authority (typically by the adoption of laws protecting intellectual property) or by technical means (e.g., cable broadcast are encrypted, so-called "unrippable" CDs have recently appeared). However, complete excludability seems hard to achieve: simply specifying intellectual property laws does not ensure that they will be enforced; similarly, technical protective measures are often imperfect and can be "cracked". As a result, *piracy* (or illicit copying) cannot be completely evacuated.

Over the last decade, the fast penetration of the Internet and the increased digitization of information have turned piracy of information goods (in particular music, movies and software) into a topic of intense debate. A selection of news headlines gathered over the last two months (February-March 2002) illustrates the current extent of the debate. These headlines are about (i) a proposed anti-piracy bill in the US that would ultimately require computer and consumer electronics companies to build piracy-prevention software into their products, (ii) a man facing jail in California for Web sales of CDs, (iii) the release of new peer-to-peer file-sharing softwares aiming to replace Napster, (iv) music distributors estimating that retail sales may be down as much as 10 percent during the past year as consumers shift to new technologies like copying CDs and downloading songs, (v) music companies settling a lawsuit with a CD consumer who alleged that the CD she purchased did not meet consumer expectations because it could not be played on a computer, or (vi) a Taiwanese Web site that offers access to a huge library of films for just \$1 each (and which,

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<sup>1</sup>Nonrivalness in consumption is usually defined by saying that the consumption possibilities of one individual do not depend on the quantities consumed by others. This is equivalent to say that, for any given level of production, the marginal cost of providing the good to an additional consumer is zero.

understandably, has drawn Hollywood's ire).<sup>2</sup>

Not surprisingly, economists have recently shown a renewed interest in information goods piracy. Here follows a selection of recent working papers which investigate a number of topical issues. Gayer and Shy (2001a) show the inefficiency of using hardware taxation to compensate copyright owners for infringements of their intellectual property (IP). In another article (Gayer and Shy, 2001b), the same authors investigate how producers of digital information goods can utilize the Internet's distribution channels, such as peer-to-peer systems, to enhance sales of their goods sold in store. The welfare implications of peer-to-peer distribution technologies are also the concern of Duchêne and Waelbroeck (2001); they show that the losses generated by illegal copies can be offset by the introduction of new products, which creates a positive surplus for their creators, as well as consumers. The idea that copyright infringement could be strategically promoted by creators is also explored by Ben-Shahar and Jacob (2001); they show, in a dynamic model, that creators might favor selective copyright enforcement as a form of predatory pricing in order to raise barriers to entry. Turning to policy matters, Harbaugh and Khemka (2001) argue that copyright enforcement targeted at high-value buyers raises copyright holder profits but, at the same time, increases piracy relative to no enforcement; therefore, they contend that either no enforcement or relatively extensive enforcement is the best policy against Internet piracy. In the same vein, Chen and Png (2001) examine how the government should set the fine for copying, tax on copying medium, and subsidy on legitimate purchases, while a monopoly publisher sets price and spending on detection; they conclude that government policies focussing on penalties alone would miss the social welfare optimum. Finally, Hui, Png and Cui (2001) provide one of the rare attempts to estimate empirically the actual impact of piracy on the legitimate demand for information goods. Using international panel data for music CDs and cassettes, they find that the demand for both goods decreased with piracy.

These recent contributions revive the literature on the economics of copying and copyright, which was initiated some twenty years ago. The seminal papers discussed the effects of photocopying and examined, among other things, how publishers can appropriate indirectly some revenues from illegitimate users (Novos and Waldman, 1984, Liebowitz, 1985, Johnson, 1985, and Besen and Kirby, 1989). The economics of IP protection was then addressed more gener-

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<sup>2</sup>Sources: (i) *Proposed anti-piracy bill draws fire*, by Stefanie Olsen (CNET News.com, March 25, 2002), (ii) *Man faces jail for Web sales of CDs*, by Lisa M. Bowman (CNET News.com, March 22, 2002), (iii) *Goodbye Napster, Hello Morpheus (and Audiogalaxy and Kazaa and Grokster...)*, by Erick Schonfeld (Business2.com, March 15, 2002), (iv) *Digital Music Fight Traps Retailers*, by Benny Evangelista (Newsfactor.com, March 12, 2002), (v) *Consumer claims victory in CD lawsuit*, by Lisa M. Bowman (CNET News.com, February 22, 2002), (vi) *Plug pulled on site selling \$1 movies*, by John Borland (CNET News.com, February 19, 2002).

ally by Landes and Posner (1989) and Besen and Raskind (1991). Both papers discuss the following trade-off between *ex ante* and *ex post* efficiency considerations. From an *ex ante* point of view, IP protection preserves the incentive to create information goods, which (as argued above) are inherently public (absent appropriate protection, creators might not be able to recoup their potentially high initial creation costs). On the other hand, IP rights encompass various potential inefficiencies from an *ex post* point of view (protection grants de facto monopoly rights, which generates the standard deadweight losses; also, by inhibiting imitation, IP rights might limit the creators' ability to borrow from, or build upon, earlier works, and thereby increase the cost of producing new ideas). A third wave of papers paid closer attention to software markets and introduced network effects in the analysis. Conner and Rumelt (1991), Takeyama (1994), and Shy and Thisse (1999) share the following argument: because piracy enlarges the installed base of users, it generates network effects that increase the legitimate users' willingness to pay for the software and, thereby, potentially raises the producer's profits. Finally, and closer from us, Watt (2000) has surveyed—and extensively supplemented—the literature on the economics of copyright.

The aim of the present paper is to address several of the themes studied so far in the literature within a simple and unified model. As a number of recent papers, we use the framework proposed by Mussa and Rosen (1978) for modelling vertical (quality) differentiation: copies are seen as lower-quality alternatives to originals (i.e., if copies and originals were priced the same, all consumers would prefer originals). In a benchmark model, we consider the market for a single information good. A monopolist must set the price for the original good, taking into account that consumers can alternatively acquire a lower-quality copy at a constant cost. The optimal strategy for the monopolist can usefully be described by using Bain (1956)'s taxonomy of an incumbent's behaviour in the face of an entry threat. Unless the quality/price ratio of copies is very low (meaning that piracy exerts no threat and will therefore be 'blockaded'), the producer will have to modify his behavior and decide whether to set a price low enough to 'deter' piracy, or to 'accommodate' piracy and make up for it by extracting a higher margin from fewer consumers of originals. Whatever the producer's optimal decision, we show that piracy reduces the producer's profits but increases consumers surplus more than proportionally: as a result, piracy (which amounts here to the provision of a cheaper and lower-quality alternative to a monopolized good) enhances social welfare.

The previous conclusion simply restates the *ex post* efficiency consideration of the traditional economic analysis of copying: if the information good was (legally or technically) better protected, the producer would fully enjoy his monopoly position and social welfare would be reduced. As argued above, such *ex post* inefficiency has to be balanced against *ex ante* considerations relating

to creation costs. To incorporate this dimension, we extend the benchmark model by considering an arbitrary number of information goods. The Mussa-Rosen framework continues to apply for each information good. Moreover, to focus on the effects of piracy, we assume that copying is the only source of interdependence between the demands for the various information goods. In particular, the goods are completely differentiated and consumers are assumed to have a sufficient (exogenous) budget to buy them all if they wish so.

Whether demands are interdependent or not depends on the nature of copying technology. In the spirit of Johnson (1985), we examine two extreme scenarios: the copying technology involves either a constant unit cost and no fixed cost, or a positive fixed cost and no marginal cost. In the former case, demands for originals are completely independent of one another: all producers act thus like the single-good monopolist of the benchmark model. Assuming a fixed creation cost that varies through producers, we can derive the number of information goods that are created at the long-run, free-entry, equilibrium. Obviously, piracy reduces this number. We can then balance *ex ante* and *ex post* efficiency considerations and show that piracy is likely to damage welfare in the long run (unless copies are a poor alternative to originals and/or are expensive to acquire).

The tractability of the model with variable-copying costs allows us to enrich the welfare analysis by introducing network externalities and a peer-to-peer technology (the first extension applies more to software and the second, to music). In contrast with the afore-mentioned papers which argue that piracy could benefit producers of originals in the presence of network externalities, we show that no such optimistic conclusion holds here. Network externalities do not resolve the trade-off between *ex ante* and *ex post* efficiency considerations; they seem, however, to attenuate slightly the negative long-run welfare impact of piracy. As for the peer-to-peer technology, our analysis suggests that its effects on consumers are ambiguous. We model the peer-to-peer technology by positing a negative relationship between the marginal cost of copying and the number of illegitimate users (or ‘pirates’). Surprisingly, as this relationship intensifies (i.e., as the peer-to-peer technology becomes more performant), the number of illegitimate users might decrease: lower copying costs induce producers to reduce the price of originals and this reduction may well overcome the decrease in the copying cost, meaning that less users decide to make copies.

The picture changes dramatically when the copying technology involves only a positive fixed cost. The demands for originals now become interdependent because consumers base their decision to invest in the copying technology on the cost of this technology and on the prices of *all* originals. Therefore, piracy introduces strategic interaction between the producers of originals whom everything else otherwise separates. This strategic interaction makes the producers’ pricing behavior (which takes the form of a simultaneous Bertrand game) more

interesting—but also much more intricate—to analyze. Due to the complexity of the system of demands, we are unable to provide a complete characterization of the set of Bertrand-Nash equilibria. We shed, nevertheless, some light on symmetric equilibria in which piracy is either blockaded, deterred or accommodated. We show, in particular, that the latter two equilibria rely on a set of rather restrictive conditions, as the incentives for unilateral deviation are high: producers tend to free-ride (by setting higher prices) when it comes to deter piracy, or they tend to undercut when it comes to accommodate piracy.

The rest of the paper is organized as follows. In Section 2, we lay out a benchmark model with a single information good and we analyze the short-run welfare effects of piracy. Then, we extend the benchmark model towards a multi-good setting in two different ways. In Section 3, we assume that copying involves a constant marginal cost and no fixed cost. Under this assumption, we examine the long-run welfare effects of piracy; we also enrich the analysis by introducing network externalities and peer-to-peer systems. In Section 4, we assume instead that copying involves a positive fixed cost and no marginal cost. Due to the intricacies of the model under this alternative assumption, we leave welfare considerations aside and try instead to unravel the complex situation of strategic interaction that piracy induces between producers of originals. We conclude and propose an agenda for future research in Section 5. Finally, we provide the proofs of the main propositions in Section 6.

## 2 A simple single good model

We start by considering a very simple market for an information good supplied by a single producer. We use the framework proposed by Mussa and Rosen (1978) for modelling vertical (quality) differentiation. There is a continuum of potential users who are characterized by their valuation,  $\theta$ , for the information good. We assume that  $\theta$  is uniformly distributed on the interval  $[0, 1]$ . Each user can obtain the information good in two different ways. One possibility is to *buy* the legitimate product (an “original”) at price  $p$ . Originals are produced by a single producer at zero marginal cost. The alternative is to *copy* the product at a cost  $c \geq 0$ . (In both cases, each user consumes at most one unit of the information good.) The two variants of the information good are indexed by their quality: let  $s_o > 0$  denote the quality of an original and  $s_c$  (with  $0 < s_c < s_o$ ), the quality of a copy.

The cost  $c$  can be thought of as the cost of the copying medium.<sup>3</sup> The assumption that the quality of a copy is lower than the quality of an original ( $s_c < s_o$ ) is common (see, e.g., Gayer and Shy, 2001a) and may be justified in several ways. In the case of analog reproduction, copies represent poor substitutes to originals. For instance, even the best photocopying loses information

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<sup>3</sup>See the end of the present section for a discussion on the precise nature of this cost.

such as fine lines, fine print and true color images. Furthermore, copies of analog media are rather costly to distribute. Although this is no longer true for digital reproduction, originals might still provide users with a higher level of services, insofar as that they are bundled with valuable complementary products which can hardly be obtained otherwise.<sup>4</sup> Finally, the user who illegally copies the product might be detected and then will be deprived of the copy. If we let  $\rho \equiv (s_o - s_c)/s_o$  denote the probability of being detected, then consumer  $\theta$ 's utility from a copy,  $\theta s_c$ , can be understood as the expected utility of enjoying the quality of an original without being detected,  $(1 - \rho) \theta s_o$ .

Accordingly, a user indexed by  $\theta$  has a utility function defined by

$$U_\theta = \begin{cases} \theta s_o - p & \text{if buying an original,} \\ \theta s_c - c & \text{if making a copy,} \\ 0 & \text{if not using the information good.} \end{cases} \quad (1)$$

We assume that  $c < s_c$ , so that the user with the highest valuation for the product is better off making a copy than not using the product (otherwise, piracy would trivially not be an issue).

## 2.1 User behaviour

A user indexed by  $\theta$  will buy the legitimate product under the following two conditions. First, buying must provide a higher utility than not using:  $\theta s_o \geq p$ . Second, buying must provide a higher utility than copying:  $\theta s_o - p \geq \theta s_c - c$ , which is equivalent to

$$\theta \geq \theta_1 \equiv \frac{p - c}{s_o - s_c}.$$

On the other hand, the same user will copy the product if the previous condition is reversed ( $\theta < \theta_1$ ) and if copying provides a higher utility than not using:  $\theta s_c - c \geq 0$ , or

$$\theta \geq \theta_2 \equiv \frac{c}{s_c}.$$

According to the value of  $p$ , three demand patterns might emerge. First, if the price of the legitimate product is too high (more precisely, if  $p \geq s_o - s_c + c$ ), then no user will buy the legitimate product: users indexed on  $[\theta_2, 1]$  copy the product whilst others do not use. Since the producer would make no profit in such a case, we can safely ignore it. Let us examine the demand pattern in the other two cases, with or without piracy. We use the following notation. Let  $L(p)$ ,  $I(p)$  and  $N(p)$  denote, respectively, the demand for the legitimate product, the demand for copies, and total demand.

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<sup>4</sup>For instance, many pieces of software come with free manuals and supporting services, or with discount on upgrades, all advantages that users who pirate the software will have to acquire at a positive price.

**Piracy exists.** For *intermediate prices* (i.e., for  $cs_o/s_c \leq p \leq s_o - s_c + c$ ), users indexed on  $[\theta_1, 1]$  buy the legitimate product, users indexed on  $[\theta_2, \theta_1]$  copy the product, others do not use. Accordingly,

$$L(p) = 1 - \frac{p - c}{s_o - s_c}, \quad (2)$$

$$I(p) = \frac{p - c}{s_o - s_c} - \frac{c}{s_c}, \quad (3)$$

$$N(p) = 1 - \frac{c}{s_c}. \quad (4)$$

Looking at the demand for the legitimate product, we notice that  $L(p)$  shifts outward as  $c$  increases (i.e., as the “price” of substitutable copies increases). Also, as  $s_c$  decreases (i.e., as the quality of copies is degraded, e.g. through stronger copyright enforcement),  $L(p)$  becomes less elastic: the producer of the legitimate product enjoys more market power as copies become a poorer substitute to originals. Obviously, the reverse results apply for the demand for copies. Finally, noteworthy is the fact that total demand depends only on the attributes of copies (i.e.,  $c$  and  $s_c$ ): the price of the legitimate product only determine the split between legal and illegal users (this is because the marginal user is just indifferent between pirating and not using).

Defining *consumer surplus* for a category of users as the sum of their utilities, we easily get the following:

$$CL(p) = \int_{\theta_1}^1 (\theta s_o - p) d\theta = L(p) \left( s_o - p - \frac{s_o}{2} L(p) \right) \text{ for legitimate users,}$$

$$CI(p) = \int_{\theta_2}^{\theta_1} (\theta s_c - c) d\theta = \frac{s_c}{2} (I(p))^2 \text{ for illegal users.}$$

**Piracy does not exist.** For *low prices* (i.e., for  $0 \leq p \leq cs_o/s_c$ ), users indexed on  $[p/s_o, 1]$  buy the legitimate product, whilst others do not use. The following is then readily checked (with the index 0 indicating the absence of piracy):

$$I_0(p) = 0, \quad L_0(p) = N_0(p) = 1 - \frac{p}{s_o},$$

$$CI_0(p) = 0, \quad CL_0(p) = C_0(p) = \int_{p/s_o}^1 (\theta s_o - p) d\theta = \frac{1}{2s_o} (s_o - p)^2.$$

Note that in the present case, total demand depends on the price of the legitimate product since the marginal user is just indifferent between buying and not using.

The previous demand pattern would also be observed in a hypothetical economy where piracy would not be an option and where users would simply decide to buy the information good or not. Keeping this reference in



mind, it is worth comparing the two demand patterns and draw some primary conclusions about the effect of piracy. Easy computations establish that  $L(p) + (s_c/s_o) I(p) = 1 - (p/s_o)$ , which implies

$$\frac{L_0(p) - L(p)}{I(p)} = \frac{s_c}{s_o} < 1.$$

In words, for a given price of the information good, the ratio between the number of lost buyers due to piracy and the number of illegal users is less than unity. Therefore, it would be wrong in these circumstances to base an estimate of the losses resulting from piracy on the assumption that all illegal users would necessarily become buyers if piracy were infeasible.

We can also have a first idea about the effect of piracy on consumer surplus. From the fact that  $s_o L(p) = s_o - p - s_c I(p)$ , it follows that

$$\begin{aligned} CL_0(p) - CL(p) &= (s_c/s_o) CI(p), \\ C(p) &= C_0(p) + \frac{s_o - s_c}{s_o} CI(p). \end{aligned}$$

The first result says that, for a given price of the information good, piracy reduces the surplus of legitimate consumers by an amount equal to a proportion  $(s_c/s_o)$  of the surplus of illegal users. According to the second result, piracy *increases* total consumer surplus by an amount equal to a proportion  $(s_o - s_c)/s_o$  of the surplus of illegal users. This means that the decrease in the legitimate users' surplus is more than compensated by the creation of the illegal users' surplus.

It must be kept in mind that the previous comparisons are only indicative. They tell us how piracy affects the demand for the legitimate product and the consumer surplus, *under the assumption that the price remains the same with or without piracy*. However, this assumption generally does not hold. As we will now show, the producer of the legitimate product modifies his pricing behaviour when he faces copying.

## 2.2 Producer's behaviour

The producer's problem is to choose the price  $p$  of the legitimate product so as to maximize profits,  $pL(p)$ , with demand given by

$$D(p) = \begin{cases} 0 & \text{for } p \geq s_o - s_c + c, \\ 1 - \frac{p-c}{s_o-s_c} & \text{for } \frac{cs_o}{s_c} \leq p \leq s_o - s_c + c, \\ 1 - \frac{p}{s_o} & \text{for } 0 \leq p \leq \frac{cs_o}{s_c}. \end{cases} \quad (5)$$

The producer's problem is complicated by the fact that some users are better off copying the product once the price exceeds some threshold. There is thus a kink in the demand curve and the producer has to choose in which segment of the demand curve to operate. By analogy with Bain (1956)'s taxonomy of

an incumbent's behaviour in the face of an entry threat, we will say that the producer is either able to 'blockade' piracy, or that he must decide whether to 'deter' piracy or 'accommodate' it. Let us now define and compare these three options.

**The producer blockades or deters piracy.** By setting a price sufficiently low, the producer can eliminate piracy. The producer's maximization program is then

$$\max_p \pi(p) = pL_0(p) = p \left(1 - \frac{p}{s_o}\right) \text{ s.t. } p \leq \frac{cs_o}{s_c}.$$

The unconstrained profit-maximizing price and profits are easily computed as

$$p_b = \frac{s_o}{2}, \quad \pi_b = \frac{s_o}{4}.$$

This solution meets the constraints if and only if  $c \geq s_c/2$ . In this case, we can say that piracy is actually *blockaded*: the producer safely sets his price as if copying was not a threat. Otherwise, piracy cannot be blockaded but the producer modifies his behaviour to successfully *deter* piracy: he will chose the highest price compatible with the constraints, i.e.

$$p_d = \frac{cs_o}{s_c}, \text{ which implies } \pi_d = \frac{cs_o(s_c - c)}{s_c^2}.$$

**The producer accommodates piracy.** The other option is to set a higher price and tolerate piracy. The producer's program becomes

$$\max_p \pi(p) = pL(p) = p \left(1 - \frac{p - c}{s_o - s_c}\right) \text{ s.t. } \frac{cs_o}{s_c} \leq p \leq s_o - s_c + c.$$

Here, the unconstrained profit-maximizing price is equal to

$$p_a = \frac{s_o - s_c + c}{2}, \text{ which implie } \pi_a = \frac{(s_o - s_c + c)^2}{4(s_o - s_c)}.$$

This solution satisfies the constraints if and only if

$$\frac{s_o - s_c + c}{2} \geq \frac{cs_o}{s_c} \iff c \leq \frac{s_c(s_o - s_c)}{2s_o - s_c}.$$

If the latter condition is not met, it is easily checked that the corner solution is equivalent to piracy deterrence.

**Blockade, deter or accommodate?** Collecting the previous results, we observe that the producer's optimal strategy depends on the relative attractiveness of copies (i.e., on the values of  $c$  and  $s_c$ ), as summarized in Proposition 1 and illustrated in Figure 1 (which is drawn for  $s_o = 1$ ).

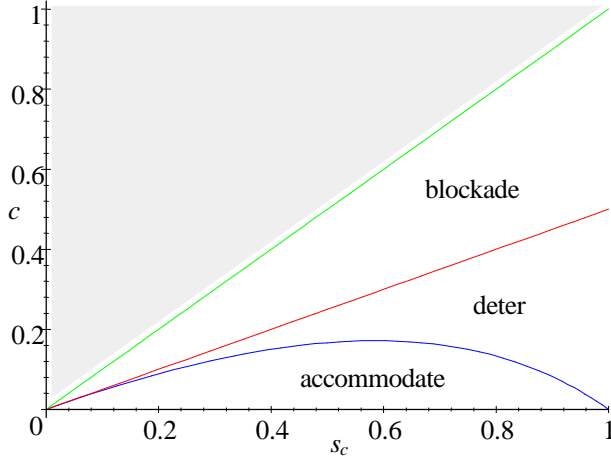


Figure 1: Producer's attitude towards piracy

**Proposition 1** *The producer's profit-maximization price is*

$$\begin{cases}
 p_b = \frac{s_o}{2}, & \text{for } \frac{s_c}{2} \leq c \leq s_c & (\text{piracy is blockaded}), \\
 p_d = \frac{cs_o}{s_c}, & \text{for } \frac{s_c(s_o - s_c)}{2s_o - s_c} \leq c \leq \frac{s_c}{2} & (\text{piracy is deterred}), \\
 p_a = \frac{s_o - s_c + c}{2}, & \text{for } 0 \leq c \leq \frac{s_c(s_o - s_c)}{2s_o - s_c} & (\text{piracy is accommodated}).
 \end{cases}$$

### 2.3 Welfare effects of piracy in the short run

Now that we have characterized the producer's pricing behaviour, we are in a position to refine our previous analysis of the effects of piracy. The previous analysis was carried out by referring to a hypothetical economy where piracy would be infeasible, and under the assumption that piracy did not affect the price of the legitimate product. Now, we know that piracy does affect the producer's pricing decision. We can also define precisely the notion of 'infeasible piracy': it corresponds to the case of blockaded piracy, defined by the condition  $c \geq s_c/2$ .

**Copies are relatively unattractive.** If  $s_c(s_o - s_c)/(2s_o - s_c) \leq c \leq s_c/2$ , we know that the producer prefers to *deter* piracy. In this case, the only effect of piracy is to force the producer to set a lower price than the one he would set if piracy were not a threat ( $p_d < p_b$ ). Although more users buy the legitimate product ( $L_d > L_b$ ), the producer's profit falls, meaning that piracy hurts him ( $\pi_d < \pi_b$ ). However, the consumer surplus clearly increases and this increase offsets the reduction in profit, which results in an increase in

social welfare (computed as the sum of consumer surplus and producer's profit:  $W_d = s_o (s_c^2 - c^2) / 2s_c^2 > W_b = 3s_o/8$ ). The possibility of making copies can be seen as a potential competition that disciplines the producer of the legitimate product in a welfare-enhancing way.

**Copies are relatively attractive.** For lower values of  $c$  (i.e.,  $c \leq s_c (s_o - s_c) / (2s_o - s_c)$ ), piracy is *accommodated*. The welfare analysis becomes a bit more complicated and also more instructive. There are now users who get a positive surplus by copying the legitimate product and they have to be taken into account in the welfare analysis. Consider first the producer. Being just a threat (as in the previous case) or an actual fact (as here), piracy has the same effect on the producer's pricing behaviour: price has to go down (though less than under the deterrence option,  $p_d < p_a < p_b$ ) and the increased demand this generates ( $L_a > L_b$ ) is not enough to prevent profit from falling ( $\pi_a < \pi_b$ ). So, as in the previous case, the producer of the legitimate product suffers from piracy.

An interesting (and much debated) question concerns the *estimation of the producer's losses*. We suggested above that these losses would be over-estimated were they computed as the potential revenue the producer would get if all illegal users bought the product instead of copying it. This claim was perfectly correct under the assumption that the feasibility of piracy had no effect on the price of the legitimate product. However, we have now shown that the latter assumption does not hold: piracy induces a decrease in price. As a result, we might just reach the opposite conclusion. For instance, if  $s_c > s_o/2$ , the producer's losses are *under-estimated* when they are computed by multiplying the number of illegal users by the current price: even if all illegal users were also buying the legitimate product at the current price, the producer would still fall short of the profit level  $\pi_b$  he could reach if piracy were infeasible.<sup>5</sup>

It can be argued that, from a social point of view, there is no reason to worry about the previous result: piracy has the advantage of breaking down the monopoly the producer would enjoy otherwise. As we have just shown, piracy leads to a lower price and a higher quantity consumed: legitimate users enjoy thus a larger surplus. Moreover, if we also incorporate the surplus enjoyed by illegal users, we find again that *piracy has a positive impact on welfare*. Computing social welfare when piracy is accommodated as  $W_a = \pi_a + CL_a + CI_a$ , we observe that

$$W_a - W_b = \frac{1}{8} \frac{(4s_o - s_c)c^2 + s_c(s_c - 2c)(s_o - s_c)}{s_c(s_o - s_c)} > 0. \quad (6)$$

We record the above two findings in the following proposition.

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<sup>5</sup>To have  $\pi_b - \pi_a < p_a I_a$  (over-estimation), we need  $s_c < s_o/2$  and  $c < (1/2) \left( \sqrt{s_o(s_o - 2s_c)} - (s_o - 2s_c) \right)$ .

**Proposition 2** *As long as it cannot be blockaded, piracy improves social welfare in the short run.*

The intuition underlying Proposition 2 is obvious. By introducing a cheaper imperfect substitute for originals, piracy reduces the monopoly power of the producer and, thereby, increases social welfare. This result must, however, be qualified in one important way. Most generally, the creation of information goods involves substantive fixed “first-copy” costs. So far, we have abstracted this fixed cost away by assuming implicitly that the producer could cover it even when he had to accommodate piracy. That is, noting the fixed creation cost by  $F$ , we have assumed that  $\pi_a > F$ . It is only under that assumption that the above result holds. Indeed, if we had instead that  $\pi_b > F > \pi_a$ , the producer would not create the information good if he had no other choice than to accommodate piracy. In such a case, piracy would clearly reduce social welfare.

**Long-run perspective.** In the next two sections, we examine the previous issue more closely by considering a multi-product framework. More precisely, we extend the benchmark model by assuming that users now have the possibility to consume from a set  $S$  of information goods (with  $|S| \geq 2$ ). As before, consumers choose, for each product, to either buy an original, make a copy, or not consume at all. We make the following assumptions about these three possibilities.

- *No use.* As before, the utility from not consuming any variant of a product is normalized to zero.
- *Originals.* Each original is produced by a separate producer, at zero marginal cost. All originals are assumed to be (i) of the same quality (indexed by  $s_o > 0$ ) and (ii) perfectly (horizontally) differentiated. Hence, if consumer  $\theta$  buys a unit of each product in the subset  $M \subseteq S$ , her utility is given by  $m\theta s_o - \sum_{i \in M} p_i$ , where  $m = |M|$  and  $p_i$  is the price charged for product  $i$ .
- *Copies.* As for originals, all copies are assumed to be (i) of the same quality (indexed by  $0 < s_c < s_o$ ) and (ii) perfectly (horizontally) differentiated. Regarding their cost, we consider, in the spirit of Johnson (1985), two extreme scenarios: the copying technology involves either a constant unit cost ( $c > 0$ ) and no fixed cost (“*variable copying cost*” model), or a fixed cost ( $C > 0$ ) and no marginal cost (“*fixed copying cost*” model). Supposing that consumer  $\theta$  copies a unit of each product in the subset  $M \subseteq S$ , her utility is given by  $m(\theta s_c - c)$  in the variable copying cost model, and by  $m\theta s_c - C$  in the fixed copying cost model.

It is important to note that, in order to focus on the effects of piracy, we assume that copying is the only potential source of interdependence between the demands for the various information goods: as just mentioned, the goods are completely differentiated; moreover, we have implicitly assumed that consumers have a sufficient (exogenous) budget to buy all information goods if they wish so.

We examine the variable- and fixed copying cost models in turn. As will become apparent, the two models lead to very different results. In the former model, the demands for any particular original are completely independent from one another; we can therefore replicate the analysis of the single-product model. On the other hand, as noted by Johnson (1985), the fixed cost of the copying technology introduces some interdependence between the demands for originals: consumers will indeed base their decision to invest in the copying technology on the cost of this technology and on the prices of *all* originals.

### 3 Multiple goods and variable copying costs

We first analyze the pricing game between an arbitrary number of producers. As will be shown, with perfectly differentiated information goods and variable copying costs, the analysis remains very simple. That allows us to analyze the entry game by incorporating a fixed creation cost. Considering the equilibrium of this two-stage game, we gauge the (long run) welfare implications of piracy. Moreover, the tractability of the pricing game also permits to extend the analysis in two directions: first, we assume that the various information goods exhibit network externalities; second, we assume that the copying technology is based on a peer-to-peer system.

#### 3.1 Pricing game

When the copying technology involves a constant unit cost per copy, it is easily seen that the producers of originals act independently of one another, in accordance with the optimal behaviour derived in the single-good model. To see this more clearly, let us define the condition for a typical consumer to buy an original of good  $i$ :

$$\begin{aligned} \text{consumer } \theta \text{ buys good } i \in S &\iff \\ \theta s_o - p_i + \sum_{j \neq i} \max\{\theta s_o - p_j, \theta s_c - c, 0\} &\geq \\ \max\{\theta s_c - c, 0\} + \sum_{j \neq i} \max\{\theta s_o - p_j, \theta s_c - c, 0\}. & \end{aligned}$$

In words, the condition says that consumer  $\theta$  must be better off purchasing good  $i$  (and choosing whichever use is the most profitable for the other goods) than copying or not using good  $i$  (and still choosing whichever use is the most profitable for the other goods). Because originals are perfectly differentiated

and because each copy of an additional good costs the same constant amount, the “whichever use is the most profitable for the other goods” does not depend on which use is made of good  $i$ . Therefore, the above condition boils down to  $\theta s_o - p_i \geq \max\{\theta s_c - c, 0\}$ , which generates the same demand schedule as in the single-good model, as given by expression (5). It follows that, because all producers set the same price, users decide either to buy all information goods or to copy them all (or not to use any). To ease the exposition (and without loss of generality), we set  $s_o = 1$  for the rest of this section.

### 3.2 Entry game

Now, let  $F_i$  denote the fixed creation cost faced by producer  $i$ . We assume that the cost of creation differs among producers (some producers are more efficient at creating equivalent works than others). Specifically, we assume that  $F_i$  is drawn from some cumulative distribution function  $G(F)$ . This function is assumed to be smooth and increasing on the interval  $[\phi, \phi + 1]$ , with  $0 < \phi < \pi_b = \frac{1}{4} < \phi + 1$ ,  $G(\phi) = 0$  and  $G(\phi + 1) = 1$ . For given gross profits  $\pi$ , only the producers with  $F_i \leq \pi$  will create their information good. Hence, the total number of works created,  $n(\pi)$ , is endogenously determined as

$$n(\pi) = \begin{cases} G(\pi) & \text{if } \pi \geq \phi, \\ 0 & \text{otherwise.} \end{cases}$$

Clearly,  $n(\pi)$  is an increasing function of  $\pi$ . Therefore, we now have a better picture of the social trade-off that piracy induces: on the one hand, piracy increases social welfare per work (as demonstrated by (6) above) but, on the other hand, piracy reduces profits per work and, thereby, the number of works created.

### 3.3 Welfare effects of piracy in the long-run

We now investigate how these two effects balance when piracy is either deterred or accommodated. Global welfare (noted  $\pi_k$ ) is now defined as welfare per work (i.e., producer’s net profit plus consumer surplus) multiplied by the number of works created (if any). To ease the computations, we make the simplifying assumptions that the fixed cost of creation is distributed uniformly and that there is a unit mass of potential producers. We have thus that  $n(\pi) = \max\{\pi - \phi, 0\}$ . Global welfare is then computed as (with  $k = b, d, a$ ):

$$\begin{aligned} \pi_k &= \max \left\{ \int_{\phi}^{\pi_k} (C_k + \pi_k - F) dF, 0 \right\} \\ &= \max \left\{ \frac{1}{2} (\pi_k - \phi) (2C_k + \pi_k - \phi), 0 \right\}. \end{aligned}$$

**Piracy is deterred.** In the region of parameters where piracy is deterred, we find that the difference  $a - b$  is equivalent in sign to  $4c^2 - 2s_c(3 - 2\phi) + s_c^2(1 + 2\phi)$ . Solving this polynomial for  $c$ , we find two positive roots for all admissible values of  $s_c$  and  $\phi$ , the large root being larger than  $s_c/2$  (above which piracy is blockaded). We can therefore conclude that piracy deterrence improves global welfare when  $c$  is larger than some threshold,  $c_d(s_c, \phi)$ , which increases with  $s_c$  and  $\phi$ .<sup>6</sup>

**Piracy is accommodated.** A similar conclusion is drawn in the region of parameters where piracy is accommodated. A few lines of computations establish that *it is only when the relative quality of copies is low enough that piracy improves global welfare*. More precisely, for having  $a > b$ ,  $c$  must be larger than some threshold value,  $c_a(s_c, \phi)$ , which increases with  $s_c$  and  $\phi$ . Moreover, if  $s_c$  is larger than some lower bound (which decreases with  $\phi$ ),  $c_a(s_c, \phi)$  is larger than the boundary  $s_c(1 - s_c)/(2 - s_c)$  and, therefore, cannot be reached.<sup>7</sup>

The previous results are loosely recorded in the next proposition.

**Proposition 3** *When copying involves a constant unit cost and no fixed cost, piracy (be it accommodated or deterred) damages welfare in the long run, unless copies are a poor alternative to originals and/or are expensive to acquire.*

The intuition behind Proposition 3 goes as follows. When copies are a poor alternative to originals and/or are expensive to acquire, (actual or threatening) piracy erodes only slightly the monopoly power of the producers. Hence, there is only a small reduction in the number of works created, which is more than compensated by the increase in the consumer surplus per work. Yet, the opposite prevails as soon as copies become more attractive. Figure 2 illustrates these results. In areas  $D_1$ ,  $D_2$  and  $D_3$ , producers limit-price to deter piracy. Piracy deterrence improves global welfare in area  $D_1$ , but deteriorates it in areas  $D_2$  and  $D_3$  (worse, in area  $D_3$ , the supply of creative works is zero when piracy has to be deterred). In areas  $A_1$ ,  $A_2$  and  $A_3$ , producers accommodate piracy. Similarly, piracy accommodation improves welfare in area  $A_1$ , but deteriorates it in areas  $A_2$  and  $A_3$  (with no work created in area  $A_3$ ). Figure 2 is drawn for  $\phi = 0.05$ ; if we increase  $\phi$ , the curves separating areas  $A_i$  and  $D_i$  shift up, which reduces the region of parameters where piracy has a positive long-run effect on welfare.

### 3.4 Network externalities

Several authors have discussed unauthorized copying of software in the presence of network externalities. In contrast with the literature on IP protection,

<sup>6</sup>Formally,  $a > b$  if and only if  $c > c_d(s_c, \phi) \equiv (s_c/4) \left( 3 - 2\phi - \sqrt{5 - 20\phi + 4\phi^2} \right)$ .

<sup>7</sup>Formally,  $c_a(s_c, \phi)$  solves  $(2 - s_c)c^4 + 4(1 - s_c)^2c^3 + 2(1 - 3s_c + 3s_c^2 - (4 - s_c)\phi)(1 - s_c)c^2 + 4l(1 - s_c)^2(s_c + \phi)c - s_c^2(1 - s_c)^2(s_c + 2\phi) = 0$ .



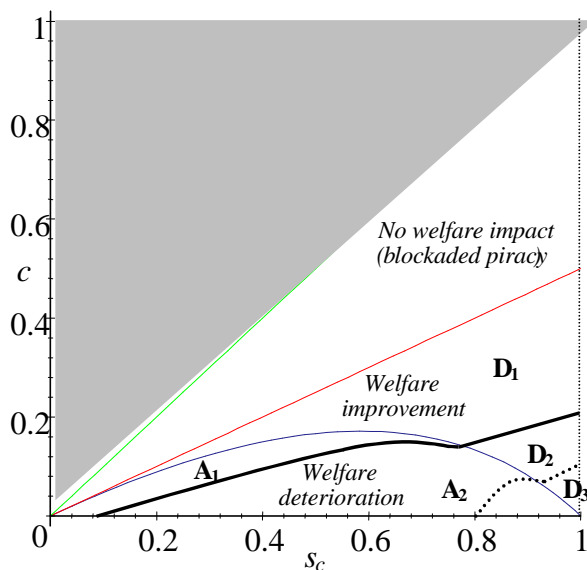


Figure 2: Long-run social effects of piracy (variable-copying cost model)

these authors have focused on the positive implications piracy might have on profits and welfare.<sup>8</sup> For instance, Takeyama (1994) demonstrates that with network externalities unauthorized reproduction of IP can not only produce greater firm profits but also lead to a Pareto improvement, compared with the case of no copying. As Takeyama explains, in the presence of network externalities, the producer has a greater incentive (*ceteris paribus*) to expand output because marginal revenue is higher. With copying, this can be achieved by the existence of low demand users who pirate (at zero cost to the firm), while high demand users purchase originals at a price which can be set at a higher level in order to incorporate the externality of increased network size created by copiers. Without copying, the same network size may only be obtained at a possibly lower price (and certainly positive marginal cost) *on all existing units*.

We now extend the benchmark model by considering information goods (such as software) which exhibit network externalities. Our objective is twofold: first, we want to examine how network externalities affect our previous results; second, we want to question the idea that piracy might improve the producer's profits when network externalities are present.

To incorporate network externalities, we assume that a user's valuation for the information good is the sum of a stand-alone component ( $\theta \in [0, 1]$ ) and a network component that increases with the *total* number of users ( $N$ ), in proportion to some factor  $\alpha > 0$ . As before, we assume that the quality

<sup>8</sup>See, e.g., Takeyama (1994), Conner and Rumelt (1991), Shy and Thisse (1999).

of originals and copies is respectively given by  $s_o$  and  $s_c < s_o$ . For reasons explained below, we also assume that  $\alpha < \min\{s_c, 1 - s_c\}$ . The utility function (1) is thus modified as follows:

$$U_\theta = \begin{cases} (\theta + \alpha N) s_o - p & \text{if buying an original,} \\ (\theta + \alpha N) s_c - c & \text{if making a copy,} \\ 0 & \text{if not using the information good.} \end{cases} \quad (7)$$

where  $N$  denotes the total number of users (legal *and* illegal).

Repeating the previous analysis of users' behaviour, we compute the two critical cut-off values as (using a “ $\sim$ ” to mark the difference with the benchmark model and setting  $s_o = 1$ ):

$$\tilde{\theta}_1 = \frac{p - c}{1 - s_c} - \alpha N, \text{ and } \tilde{\theta}_2 \equiv \frac{c}{s_c} - \alpha N.$$

Because we want to examine the effects of network externalities on piracy, we focus here on situations where piracy has to be accommodated. In these situations, the producer sets a price for originals such that  $0 \leq \tilde{\theta}_2 < \tilde{\theta}_1 < 1$ . Comparing with the benchmark model, we observe that the two cut-off values are reduced by the same magnitude,  $\alpha N$ , which measures the network effects. It follows that *the demand for copies* (given by  $\max\{\tilde{\theta}_1 - \tilde{\theta}_2, 0\}$ ) *is left unchanged*, whilst *the demand for the legitimate product (and, hence, total demand) shifts outward by  $\alpha N$* .

Another difference with the benchmark model is that *the market might now be completely covered*, in the sense that, when everybody uses the product, the user with the lowest valuation ( $\theta = 0$ ) might be better off copying the product than not using it. This is so if  $\tilde{\theta}_2 \leq 0$  when  $N = 1$ , i.e. if  $\alpha \geq c/s_c$ . For simplicity, we rule out that possibility and assume that network effects are not too strong, so that lowest valuation users do not use the product:  $\alpha < c/s_c$ .

Using the facts that  $\tilde{N}(p) = 1 - \tilde{\theta}_2$  and  $\tilde{L}(p) = 1 - \tilde{\theta}_1$ , we can derive the demand pattern in the presence of network externalities when piracy is accommodated. The producer's problem then becomes

$$\max_p \pi = p\tilde{L}(p) = p \frac{s_c(1 - s_c) + c(s_c - \alpha) - s_c(1 - \alpha)p}{s_c(1 - s_c)(1 - \alpha)} \text{ s.t. } \frac{c}{s_c} \leq p.$$

The profit-maximizing price is easily found as

$$\tilde{p}_a = \frac{s_c(1 - s_c) + c(s_c - \alpha)}{2s_c(1 - \alpha)},$$

which satisfies the constraint providing

$$c \leq \frac{s_c(1 - s_c)}{(2 - s_c - \alpha)}. \quad (8)$$

Our assumption that  $\alpha < \min\{s_c, 1 - s_c\}$  makes sure that condition (8) is compatible with the assumption that  $\alpha < c/s_c$ .<sup>9</sup>

<sup>9</sup>For  $\alpha \geq c/s_c$ , the market is completely covered. Hence,  $\tilde{N}(p) = 1$  and  $\tilde{L}(p) = 1 + \alpha -$

### 3.4.1 Impact of network externalities on the producer's profits

When the producer accommodates piracy and the market is not fully covered, his gross profit is computed as

$$\tilde{\pi}_a = \tilde{p}_a \tilde{L}_a = \frac{(s_c(1-s_c) + c(s_c - \alpha))^2}{4s_c^2(1-s_c)(1-\alpha)^2}.$$

It is easily verified that, as in the benchmark model, the producer's profits increase when the attractiveness of copies decreases (i.e., when  $c$  increases or when  $s_c$  decreases). The producer's profits also increase when network externalities become stronger.

Let us now compare this profit level with what the producer could earn in a hypothetical economy with no threat of piracy. In such an economy, the user indifferent between purchasing the product and not using it has a valuation  $\tilde{\theta}_3$  such that  $\tilde{\theta}_3 - p + \alpha N = 0$ . Since all users with a valuation higher than  $\tilde{\theta}_3$  will purchase the product, the number of buyers equals the total number of users and is determined by solving  $N = 1 - \tilde{\theta}_3 = 1 - (p - \alpha N)$ . Therefore, with straightforward notation,  $\tilde{N}_0(p) = \tilde{L}_0(p) = (1 - p) / (1 - \alpha)$ . Network externalities expand demand but do not affect the profit-maximization price which is still equal to  $1/2$ . It follows that profits in an economy with no piracy are given by  $\tilde{\pi}_b = 1 / (4(1 - \alpha))$ . Computing the difference between profits when piracy does not exist or when it has to be accommodated, we find

$$\tilde{\pi}_b - \tilde{\pi}_a = \frac{(s_c - \alpha)}{4s_c^2(1-s_c)(1-\alpha)^2} [(1-s_c)s_c^2 - 2s_c(1-s_c)c - (s_c - \alpha)c^2].$$

The term in square brackets determines the sign of the difference. Since it is a decreasing function of  $c$ , it reaches its lowest value when  $c$  equals its upper limit given by condition (8). Straightforward computations establish that this lowest value is positive, which allows us to state the following result.

**Proposition 4** *Piracy hurts the producer even in the presence of network externalities.*

It can be checked that this conclusion holds true when network externalities are strong enough for the market to be fully covered. Worse, it might even be that the gap between  $\tilde{\pi}_b$  and  $\tilde{\pi}_a$  widens as network externalities become stronger (this is true, for instance, when  $s_c > 1/2$ ).

The previous result contrasts with Takeyama's finding that piracy might improve the producer's position when network externalities are strong enough. The contrast is explained by the difference in the modeling frameworks. Takeyama considers only two types of users (high- and low-valuation users). When the

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$(p - c) / (1 - s_c)$ . It is easily checked that the interior solution always holds and is given by  $\tilde{p}_a = (1/2)[(1 + \alpha)(1 - s_c) + c]$ .

producer sells only to high-valuation users whether piracy is feasible or not, it is possible to find situations where piracy improves his profits: with piracy, the increase in the size of the network resulting from copies made by low-valuation users increases the willingness to pay of high-valuation users, thus enabling the producer to extract more surplus from them. However, this conclusion no longer holds in cases where the producer sells to both high- and low-valuation users whether piracy is feasible or not. The latter cases resemble what we have here since we assume a continuum of user types instead of just two.

### 3.4.2 Impact of network externalities on social welfare

We noted above that as network externalities intensify (i.e., as  $\alpha$  increases), the demand for the legitimate product expands, which allows the producer accommodating piracy to charge a higher price and make higher profits. Simple computations allows us to establish the following. Despite a higher price for the legitimate product, the number of legitimate users increases and so does their surplus. A higher price for the legitimate product also means that copies become relatively more attractive. Therefore, an increase in  $\alpha$  implies an increase in the number of illegal users and a higher surplus for this category of users. Collecting the previous results, we conclude that, *when piracy is accommodated, social welfare is an increasing function of the intensity of network externalities, both in the short and in the long run.*

It is easily checked that the latter conclusion also applies to our hypothetical economy where piracy is infeasible (or to settings where producers have the ability to blockade piracy). Next question which arises naturally is which setting network externalities impact the most. We will not attempt here to give a complete answer to this question. We just shed some light on it by carrying out a partial numerical analysis. Letting  $\phi = 0.05$  and  $\alpha = 0$  or  $0.1$ , we identify the region in the  $(s_c, c)$  plane where piracy accommodation improves long-run social welfare with respect to the case where piracy is infeasible. As illustrated in Figure 3, this region is clearly larger in the presence of network externalities (dotted curves represent the case where  $\alpha = 0$  with piracy improving welfare in region OAB; solid curves represent the case where  $\alpha = 0.1$  with piracy improving welfare in region OCD). This finding tends to indicate that in the presence of network externalities, (accommodated) piracy is more likely to be welfare-enhancing.

## 3.5 Peer-to-peer technology

It is usually argued that the development of the Internet facilitates piracy of copyrighted works. A prominent example of this tendency is the recent emergence of *peer-to-peer technology* (referred to as P2P), which is a type of transient Internet network that allows a group of computer users with the same

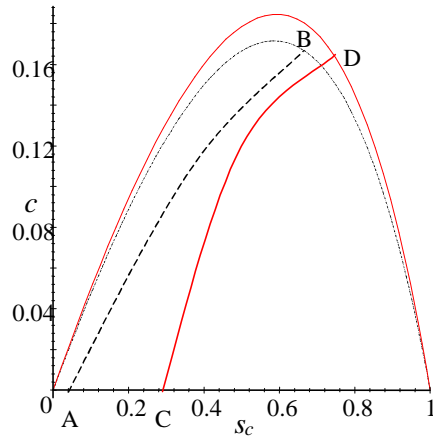


Figure 3: Effects of network externalities on global welfare

networking program to connect with each other and directly access files from one another's hard drives. Napster and Gnutella are examples of this kind of peer-to-peer software used to exchange music titles.

To account for the existence of P2P technology, we modify our benchmark model in a very simple way: we assume now that *the unit cost of making copies decreases with the number of illegal users*. We model thereby the idea that, as more users join a P2P community (e.g., Napster), it becomes less costly (e.g., in terms of Internet connection time) for each of them to find the specific titles they are looking for. Accordingly, the utility function (1) becomes:

$$U_\theta = \begin{cases} \theta s_o - p & \text{if buying an original,} \\ \theta s_c - (c - \beta I) & \text{if making a copy,} \\ 0 & \text{if not using the information good.} \end{cases} \quad (9)$$

The cost of making copies is now equal to  $c - \beta I$ : it decreases ( $\beta \geq 0$ ) with the number  $I$  of illegal users. For further reference, we say that  $\beta$  measures the “peer effect”.

### 3.5.1 Pricing game

Repeating the analysis of users' behaviour under this new assumption, we find new values for the two critical cut-offs (we now use a “^” to distinguish the variables and we still set  $s_o = 1$ ):

$$\hat{\theta}_1 = \frac{p - c + \beta I}{1 - s_c}, \text{ and } \hat{\theta}_2 \equiv \frac{c - \beta I}{s_c}.$$

Again, we focus on situations where the producer's optimal strategy is to accommodate piracy. Hence, the producer sets a price such that  $0 \leq \hat{\theta}_2 < \hat{\theta}_1 < 1$ .

1. The constraint imposed by the existence of illegal users is the same as before:  $\hat{\theta}_2 < \hat{\theta}_1$  if and only if  $p > c/s_c$ . What differs with the two previous cases is that the price level also determines whether the market is fully covered ( $\hat{\theta}_2 \leq 0$ ) or not ( $\hat{\theta}_2 > 0$ ).<sup>10</sup> We need thus to take this possibility into account when we solve the producer's problem.

Consider situations where some users purchase, others copy and others do not use. That is, suppose that  $p$  is such that  $0 < \hat{\theta}_2 < \hat{\theta}_1 < 1$ . We thus have the following equalities:  $\hat{N}(p) = 1 - \hat{\theta}_2$ ,  $\hat{L}(p) = 1 - \hat{\theta}_1$  and  $\hat{I}(p) = \hat{\theta}_1 - \hat{\theta}_2$ . Replacing  $\hat{\theta}_1$  and  $\hat{\theta}_2$  by their values, we find the following demand pattern

$$\begin{aligned}\hat{L}(p) &= \frac{s_c(1 - s_c) - \beta + cs_c - (s_c - \beta)p}{s_c(1 - s_c) - \beta}, \\ \hat{I}(p) &= \frac{s_cp - c}{s_c(1 - s_c) - \beta}, \\ \hat{N}(p) &= \frac{(1 - s_c)(s_c - c) - \beta(1 - p)}{s_c(1 - s_c) - \beta}.\end{aligned}$$

As noted above, for piracy to exist ( $\hat{I}(p) > 0$ ), it must be that  $p > c/s_c$ . Furthermore, for the market to be partially covered ( $\hat{N}(p) < 1$ ), it must also be that  $p < c(1 - s_c)/\beta$ . For  $c > 0$ , there exists a non-empty interval of prices that meet the two constraints providing that  $\beta < s_c(1 - s_c)$ , i.e. that the peer effect is not too strong. This assumption also guarantees that the own-price effect is negative for originals and for copies. We thus assume from now on that this condition is met.

To assess the impact of P2P technology on demand in these circumstances, we perform some comparative statics on the above expressions; we also compare them with the equivalent expressions (2) to (4) of the benchmark model. First, and quite obviously, we observe that the P2P technology increases the demand for copies at each value of  $s_c$ ,  $c$  and  $p$ . Indeed,  $\hat{I}(p) = \lambda I(p)$ , where  $\lambda = s_c(1 - s_c)/(s_c(1 - s_c) - \beta) > 1$ . Second, the P2P technology makes the demand for originals more elastic. As in the benchmark model, an increase in  $p$  induces users to substitute copies for originals. But the P2P technology amplifies this substitution effect since the cost of making copies goes down as the community of illegal users enlarges. This is confirmed by the following relationship:  $\hat{L}(p) = L(p) - [\beta/(1 - s_c)]\hat{I}(p)$ . Combining the previous findings, we finally observe that total demand is an increasing function of  $\beta$ :  $\hat{N}(p) = N(p) + (\beta/s_c)\hat{I}(p)$ . An important difference with the benchmark case is that the total number of users is now a function of the price of the legitimate product. The marginal user is still the one who is indifferent between copying and not using. The difference, now, is that the indirect utility from copying

<sup>10</sup>In the benchmark case, the market is never fully covered, whilst in the presence of network externalities, the market can be covered if  $\alpha > c/s_c$ . The latter condition does not depend on the price of the legitimate product and is thus beyond the producer's control.

increases with the price of originals: as this price goes up, more users make copies, which decreases the cost of piracy through the peer effect.

Let us now show that the producer will never set a price such that the market is fully covered. The highest price the producer would set is  $p_{\max}$  such that  $\hat{L}(p_{\max}) = 0$ . It is easily checked that this price lies below  $c(1 - s_c)/\beta$ , i.e., the price at which the user with the lowest valuation would be better off copying than not using. The producer maximization program is thus

$$\max_p \pi = p\hat{L}(p) = p \frac{s_c(1 - s_c) - \beta + cs_c - (s_c - \beta)p}{s_c(1 - s_c) - \beta} \text{ s.t. } p \geq \frac{c}{s_c}.$$

The unconstrained profit-maximizing price is

$$\hat{p}_a = \frac{s_c(1 - s_c) - \beta + cs_c}{2(s_c - \beta)}. \quad (10)$$

This interior solution holds if and only if  $\hat{p}_a \leq c/s_c$ , i.e., if

$$c \leq \frac{s_c(s_c(1 - s_c) - \beta)}{s_c(2 - s_c) - 2\beta}. \quad (11)$$

### 3.5.2 Impacts of the peer effect

We want now examine how a change in the peer effect (i.e., in parameter  $\beta$ ) affects the economy. We focus on the case where condition (11) is met, meaning that the producer accommodates piracy. Using expression (10), we observe that *the equilibrium price decreases as the peer effect intensifies*: as  $\beta$  increases, copies become closer substitutes to originals (because they are less expensive to make), putting the producer under more competitive pressure.

Regarding the numbers of legal and illegal users, we can hardly give any a priori answer since both originals and copies become cheaper as  $\beta$  increases. We need thus to examine the derivatives of the equilibrium numbers of users with respect to  $\beta$ . Let us first define the following thresholds:

$$c_{ill} = \frac{s_c(s_c(1 - s_c) - \beta)^2}{2\beta^2 - 2s_c(2 - s_c)\beta + s_c^2(2 - 2s_c + s_c^2)},$$

$$c_{tot} = \frac{s_c(s_c(1 - s_c) - \beta)^2}{(2 - s_c)\beta^2 - 4s_c(1 - s_c)\beta + s_c^2(1 - s_c)(2 - s_c)},$$

$$\text{with } 0 < c_{ill} < c_{tot} < \frac{s_c(s_c(1 - s_c) - \beta)}{s_c(2 - s_c) - 2\beta} \quad \forall 0 \leq s_c < 1 \text{ and } \forall 0 \leq \beta < s_c(1 - s_c).$$

We can now describe how the equilibrium number of users evolve as  $\beta$  varies:

- the number of *legitimate* users always increases with the intensity of the peer effect;

- the number of *illegal* users increases with the intensity of the peer effect if  $c < c_{ill}$  and decreases otherwise;
- the *total* number of users increases with the intensity of the peer effect if  $c < c_{tot}$  and decreases otherwise.

The intuition for these results goes as follows. The number of legitimate users increases because the decrease in the price of originals offsets the substitution effect (copies become less expensive to make). As far as illegal users are concerned, the result is ambiguous: their number increases with the intensity of the peer effect only if the cost of the copying medium is low enough. Otherwise, the decrease in the price of originals overcome the decrease in the cost of making copies and, as a result, less users decide to make copies in spite of the increase in the peer effect. Finally, the same ambiguity prevails regarding the total number of users. However, as the number of legitimate users goes up, larger values of  $c$  are required to have the total number of users decrease as the peer effect intensifies.

Let us now look at the various *surpluses*. One easily understands that, because an increase in  $\beta$  exposes the producer to more competition, it reduces the producer's profits and enlarges the legitimate users' surplus. As far as the surplus of illegal users is concerned, it can be checked that it is proportional to the square of the number of illegal users. Therefore, the above conclusion applies: the surplus of illegal users decreases with  $\beta$  for  $c$  large enough.

Finally, as long as social welfare is concerned, we investigate how the peer effect modifies our previous conclusions. Regarding the short run, it can be shown that, for all acceptable values of  $\beta$ , accommodated piracy improves social welfare with respect to the hypothetical economy where piracy is infeasible. Moreover, numerical simulations suggest that welfare increases further as the peer effect becomes stronger. On the other hand, the effect on long-run social welfare is less clear-cut: numerical simulations show here that the presence of the P2P technology narrows the region of parameters where accommodated piracy improves long-run social welfare (essentially because the region of parameters where the producers find it optimal to accommodate piracy becomes itself smaller).

## 4 Multiple goods and fixed copying cost

When copying involves a fixed cost rather than variable costs, the demands for originals become interdependent since consumers base their decision to invest in the copying technology on the cost of this technology and on the prices of *all* originals. To see the difference with the previous case, recall the condition for consumer  $\theta$  to purchase an original of good  $i$ . The condition is still that consumer  $\theta$  must be better off purchasing good  $i$  (and choosing whichever use is



the most profitable for the other goods) than copying or not using good  $i$  (and still choosing whichever use is the most profitable for the other goods). What changes is that the most profitable uses for all other goods does now depend on whether the consumer copies good  $i$  or not: if she does, then the cost of copying any number of other goods is zero instead of  $C$ .

#### 4.1 User behaviour

What is the utility consumer  $\theta$  can obtain depending on her use of good  $i$ ? Suppose that  $n \geq 2$  goods are available and let  $p_i$  denote the price of good  $i$ . Since we will be looking for symmetric Bertrand-Nash equilibria, we assume that all other goods are priced the same:  $p_j = p \forall j \neq i$ . To restrict slightly the number of cases to consider, we make the following assumptions:

$$s_c < C < ns_c, \quad (12)$$

$$ns_c > s_o. \quad (13)$$

Assumption (12) simply says that no consumer will invest in the copying technology if it is to copy only one original ( $\theta s_c - C < 0 \forall \theta$ ), but that some consumers might invest if it is to copy all  $n$  originals ( $\exists \theta$  s.t.  $\theta ns_c - C > 0$ ).<sup>11</sup> According to assumption (13), the quality differential between originals and copies is not too large (in particular,  $n$  copies are worth at least one original).

We can now determine the most profitable use for the other goods depending on the use made of good  $i$ . If the consumer either purchases or does not use good  $i$ , it is easily seen that the consumer will treat all the other goods alike: she will either buy, copy or not use them all, leaving her respectively with an additional utility of  $(n-1)(\theta s_o - p)$ ,  $(n-1)\theta s_c - C$  or 0.<sup>12</sup> On the other hand, if she copies good  $i$ , not using the other goods clearly becomes a dominated option (because the copying technology has been purchased). Hence, the consumer will either purchase or copy all other goods, leaving her respectively with an additional utility of  $(n-1)(\theta s_o - p)$  or  $(n-1)\theta s_c$ . Putting these findings together, we summarize the user's behaviour in the following lemma.

**Lemma 1** *Facing a price vector  $(p_i, (p)_{j \neq i})$  and a copying technology described*

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<sup>11</sup>The first inequality makes a clear distinction between the fixed- and variable-copying cost models. When the second inequality is reversed, piracy is trivially eliminated.

<sup>12</sup>To see this, let  $x$  (resp.  $y$ ) denote the number of goods  $j \neq i$  purchased (resp. copied), with  $0 \leq x, y \leq n-1$ . If  $x \geq 0$ , then any situation with  $0 < y < n-1-x$  is dominated. Indeed,  $y > 0$  implies that the copying technology has been purchased, meaning that copying an additional good makes the consumer strictly better off than not using this good. Now, because the utility from a purchased or a copied good is constant, any situation with  $x > 0$  and  $y = n-1-x > 0$  is also dominated. We are thus left with three possibilities: (i)  $x = n-1, y = 0$ , (ii)  $x = 0, y = n-1$ , and (iii)  $x = y = 0$ .

by (12), a consumer of type  $\theta$  purchases original  $i$  if and only if

$$\begin{aligned} \theta s_o - p_i + \max\{(n-1)(\theta s_o - p), (n-1)\theta s_c - C, 0\} \\ \geq \max\{(n-1)(\theta s_o - p), n\theta s_c - C, 0\}. \end{aligned} \quad (14)$$

The next logical step would be to use condition (14) to derive the demand for original  $i$ . Though feasible, this task turns out to be quite cumbersome. Just to give an idea of the intricacies involved, let us briefly describe the operations that separate condition (14) from a complete characterization of the demand function for original  $i$ .

1. We need first to determine the values of the two maxima for all combinations of parameters. Regarding the LHS, two patterns emerge according to whether  $p$  is below or above  $\hat{p} \equiv s_o C / ((n-1)s_c)$ . Regarding the RHS, four patterns have to be distinguished, depending on the value of  $p$  (the threshold being here  $\tilde{p} \equiv s_o C / n s_c < \hat{p}$ ) and on the value of  $s_c/s_o$  (the threshold being  $(n-1)/n$ ). Crossing these patterns, we identify six different regimes: 3 cases determined by the value of  $p$  ( $p > \hat{p}$ ,  $\tilde{p} < p \leq \hat{p}$ , and  $p < \tilde{p}$ ) times two cases determined by the value of  $s_c/s_o$  (below or above  $(n-1)/n$ ).
2. Next, in each of the six regimes, we need to compare the maxima on the two sides of the inequality. Doing so, we can define intervals for the value of  $\theta$  inside which condition (14) takes a specific form. For instance, in the regime defined by  $p > \hat{p}$  and  $s_c/s_o < (n-1)/n$ , there are five such different intervals, one of them being the following: for  $\theta \in [C/n s_c, C/((n-1)s_c)]$ , condition (14) rewrites as  $\theta s_o - p_i \geq n\theta s_c - C \iff \theta \geq (C - p_i)/(n s_o - s_c)$ .<sup>13</sup>
3. For each interval so defined, the next step consists in establishing conditions on  $p_i$  under which the specific form of condition (14) is satisfied. In the previous example, it can be checked that the condition is (i) never met in the interval if  $p_i \geq C s_o / (n s_c)$ , and (ii) always met in the interval if  $p_i \leq C (s_o - s_c) / ((n-1)s_c)$ .
4. The last operation consists in collecting all the previous results and to identify, for different ranges of  $p_i$ , the corresponding mass of consumers who prefer buying original  $i$ . The resulting demand function will typically exhibit a number of kinks (up to 5 in some regimes; see expression (19) for an illustration).

Obviously, this cumbersome process is only a preliminary step towards the characterization of the set of Nash equilibria in prices. Indeed, we should next

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<sup>13</sup>The consumers in this interval decide either to purchase and consume  $i$  only, or to copy all products.

use the demand function to maximize firm  $i$ 's profit and, thereby, derive firm  $i$ 's reaction function. It is easily understood that the combination of several demand regimes and several kinks in the demand function under each regime makes this process even more daunting than the previous one. Furthermore, because demand functions are discontinuous, payoff functions may fail to be quasi-concave, which may lead to the non-existence of an equilibrium in pure strategies (see Dasgupta and Maskin, 1986).

We thus renounce to try and give a complete characterization of symmetric Bertrand-Nash equilibria in the model with fixed copying costs. Instead, we provide conditions under which some specific equilibria might (or might not) occur. The price vectors we investigate correspond to the three patterns examined in the model with variable copying costs: blockaded, deterred and accommodated piracy.

## 4.2 Blockaded piracy

As before, piracy is blockaded if market conditions are such that piracy exerts no threat on producers of originals even when each of them behaves as an unconstrained monopolist. Because, when there is no threat of piracy, the demands for the  $n$  originals are completely independent of one another, each producer chooses  $p_i$  so as to maximize  $\pi_i = p_i(1 - p_i/s_o)$ . That is, each firm charges  $p_b = s_o/2$ . The next proposition states under which condition this behaviour constitutes a Nash equilibrium.

**Proposition 5** (Blockaded piracy) *Each firm charging the monopoly price,  $p_b = s_o/2$ , is a Nash equilibrium of the game with fixed copying costs if and only if  $C \geq (n/2)s_c$ .*

**Proof.** See the Appendix 6.1. ■

The message of Proposition 5 is clear: *if the most eager consumer needs to copy more than half of the available originals to recoup the fixed cost of the copying technology, then piracy exerts no threat on the producers of originals, who can safely charge the monopoly price.*

Let us now turn to the situations where piracy *cannot* be blockaded; that is, we assume that  $C < (n/2)s_c$ . In these situations, piracy becomes an actual threat and producers of originals have to decide whether it is more profitable for them to deter or to accommodate piracy. As already stressed, the increasing returns to scale in the copying technology transform the choice between piracy deterrence or accommodation into a problem of *interdependent* decision making.

## 4.3 Deterred piracy

To deter piracy of its product, firm  $i$  must find the 'limit price',  $\bar{p}_i$ , under which all consumers find the original product relatively more attractive than

the copy. In the simple model of Sections 2 and 3, firm  $i$  could solve this problem in total independence: for consumers to prefer copying to both purchasing and not using, it had to be the case (respectively) that  $\theta < (p - c) / (s_o - s_c)$  and  $\theta \geq c/s_c$ ; clearly, any price below  $\bar{p} = cs_o/s_c$  made the joint satisfaction of the two conditions impossible and, thereby, deterred piracy.

Now, when copying involves a fixed cost, firm  $i$ 's limit price will clearly depend on the prices set by the other firms. Intuitively, piracy should be harder to deter (in the sense that firm  $i$  will have to decrease its price further) the higher the price set by the other firms, and conversely. Indeed, if the other firms set a relatively high price, consumers will have more incentive to invest in the copying technology and, because of increasing returns to scale, they will tend to copy product  $i$  along with the other products, unless the price of  $i$  is considerably lower.

To formalize the intuition, we first determine firm  $i$ 's limit price supposing that all other firms charge the same arbitrary price  $p$ . That is, we characterize the function  $\bar{p}_i(p)$ . We then look for a fixed point of this function,  $\bar{p}$ , and determine under which conditions all firms charging  $\bar{p}$  is a Bertrand-Nash equilibrium, in which piracy is (collectively) deterred.

### 4.3.1 Individual limit pricing

Suppose  $p_j = p \forall j \neq i$ . We want to determine the limit price  $\bar{p}_i(p)$  under which no consumer finds it profitable to copy product  $i$ . Note that firm  $i$  is concerned only by deterring the copying of its own product. Yet, as we will see, its behaviour will depend on whether consumers copy or not the other products.

Using the analysis of the user behaviour (summarized in Lemma 1), let us define the utility for user  $\theta$  of, respectively, buying or copying product  $i$ :

$$\begin{aligned}
 U_B(\theta, p_i, p) &= \theta s_o - p_i + \underbrace{\max\{(n-1)(\theta s_o - p), (n-1)\theta s_c - C, 0\}}_{MB} \\
 U_C(\theta, p_i, p) &= \theta s_c - C + \underbrace{\max\{(n-1)(\theta s_o - p), (n-1)\theta s_c\}}_{MC}
 \end{aligned}$$

By comparing the exact values of  $MB$  and  $MC$ , we can express the precise form of the condition  $U_B(\theta, p_i, p) \geq U_C(\theta, p_i, p)$  for all configurations of prices and parameters. The next step consists in deriving for which values of  $p_i$  the condition is always met in the corresponding region of parameters. Straight-forward computations establish the results of this two-step procedure. Table 1 summarizes the results for the case where the other products are relatively expensive—precisely, when  $p > s_o C / [(n-1) s_c]$ . Table 2 presents the results for the other case. Finally, collecting all these results, we state firm  $i$ 's limit pricing behaviour in Lemma 2.

Users	$U_B(\cdot) \geq U_C(\cdot)$ if	Always met if
$0 \leq \theta \leq \frac{C}{(n-1)s_c}$	$\theta \leq \frac{C-p_i}{ns_c-s_0}$	$p_i \leq \frac{(s_o-s_c)C}{(n-1)s_c}$
$\frac{C}{(n-1)s_c} \leq \theta \leq \frac{(n-1)p-C}{(n-1)(s_o-s_c)}$	$\theta \geq \frac{p_i}{s_o-s_c}$	$p_i \leq \frac{(s_o-s_c)C}{(n-1)s_c}$
$\frac{(n-1)p-C}{(n-1)(s_o-s_c)} \leq \theta \leq \frac{p}{s_o-s_c}$	$\theta \geq \frac{p_i+(n-1)p-C}{n(s_o-s_c)}$	$p_i \leq p - \frac{C}{n-1}$
$\frac{p}{s_o-s_c} \leq \theta \leq 1$	$\theta \geq \frac{p_i-C}{s_o-s_c}$	$p_i \leq p + C$

Table 1: Limit pricing when other products are relatively expensive

Users	$U_B(\cdot) \geq U_C(\cdot)$ if	Always met if
$0 \leq \theta \leq \frac{p}{s_0}$	$\theta \leq \frac{C-p_i}{ns_c-s_0}$	$p_i \leq C - \frac{ns_c-s_o}{s_0}p$
$\frac{p}{s_0} \leq \theta \leq \frac{p}{s_o-s_c}$	$\theta \geq \frac{p_i+(n-1)p-C}{n(s_o-s_c)}$	$p_i \leq C - \frac{ns_c-s_o}{s_0}p$
$\frac{p}{s_o-s_c} \leq \theta \leq 1$	$\theta \geq \frac{p_i-C}{s_o-s_c}$	$p_i \leq p + C$

Table 2: Limit pricing when other products are relatively cheap

**Lemma 2** *To deter piracy of its product, firm  $i$  needs to set its price as follows:*

$$\begin{cases} p_i \leq \bar{p}_i(p) = \frac{(s_o-s_c)C}{(n-1)s_c} & \text{if } p > \frac{s_oC}{(n-1)s_c}, \\ p_i \leq \bar{p}_i(p) = C - \frac{ns_c-s_o}{s_0}p & \text{if } p \leq \frac{s_oC}{(n-1)s_c}. \end{cases}$$

**Proof.** The proof follows directly from the results summarized in Tables 1 and 2. It is readily checked that the first (resp. second) condition in the lemma is the most stringent among the conditions expressed in Table 1 (resp. in Table 2). ■

Lemma 2 confirms our intuition. When the other products are relatively expensive ( $p > s_oC/[(n-1)s_c]$ ), firm  $i$  must price much lower than the other firms ( $\bar{p}_i(p) = (s_o-s_c)C/[(n-1)s_c] < p$ ) in order to discourage piracy of its product. On the other hand, as the other products become cheaper, the constraint on  $i$ 's price relaxes: for  $p \leq s_oC/[(n-1)s_c]$ ,  $\bar{p}_i(p)$  decreases with  $p$ , and eventually becomes lower than  $p$ .

### 4.3.2 Symmetric limit pricing

The previous findings illustrate how firms tend to free-ride on each other when it comes to deter piracy. The only situation for which no free-riding is observed is when all firms charge the *symmetric limit price* defined by  $\bar{p}_i(p) = p$ , i.e.,<sup>14</sup>

$$p = p_d \equiv \frac{Cs_o}{ns_c}.$$

This symmetric limit price appears as a likely candidate for an equilibrium with deterred piracy. In the next proposition, we state the conditions under which

<sup>14</sup>To draw an analogy with the variable-copying cost model, note that, in this model, the total cost of copying  $n$  originals is equal to  $nc$ . Now, if we equate total copying cost in the two models (i.e., if we let  $C = nc$ ), we find exactly the same limit price.

this conjecture proves right. We first define the following threshold:

$$C_d \equiv \frac{n^2 s_c (s_o - s_c)}{(n+1) s_o - n s_c}.$$

Note that  $C_d < (n/2) s_c \iff n s_c > (n-1) s_o$ .

**Proposition 6** (Deterred piracy) *Each firm charging the symmetric limit price,  $p_d = (C s_o) / (n s_c)$ , is a Nash equilibrium of the game with fixed copying costs if and only if  $n s_c > (n-1) s_o$  and  $C_d \leq C \leq (n/2) s_c$ .*

**Proof.** See Appendix 6.2. ■

The intuition behind Proposition 6 goes as follows. Suppose the other firms charge the symmetric limit price and consider the “would-be pirates” (i.e., those users for whom  $n\theta s_c - C > 0$ ). We want to determine how those users maximize their utility when they do not purchase product  $i$ . Actually, their behaviour depends on the relative quality of copies. When the quality of copies is relatively low ( $n s_c \leq (n-1) s_o$ ), the would-be pirates prefer not using  $i$  and purchasing all other products, rather than copying all  $n$  products. Hence, there is no threat of piracy for product  $i$  and firm  $i$  sees no reason to limit its price. On the other hand, when the quality of copies is relatively high ( $n s_c > (n-1) s_o$ ), the would-be pirates becomes actual pirates if they decide not to purchase product  $i$ . To deter them to do so, firm  $i$  must therefore set a low enough price. How low this price should be depends on the fixed copying cost. If this cost is high ( $C > (n/2) s_c$ ), firm  $i$  can free-ride on the other firms’ effort and set the monopoly price. If the copying cost is low ( $C < C_d$ ), the opposite prevails: firm  $i$  must set a limit price below  $p_d$ . Finally, for intermediary fixed-copying costs, firm  $i$  optimally deters piracy by charging the same price as the other firms.

### 4.3.3 Welfare effects of deterred piracy

When the conditions of Proposition 6 are met and  $n$  information goods are produced, each producer makes the following (gross) profit:

$$\pi_d(n) = \left(1 - \frac{C}{n s_c}\right) \frac{C s_o}{n s_c} < \pi_b(n) = \frac{s_o}{4},$$

where  $\pi_b(n)$  is the (gross) profit that each producer could obtain if piracy did not exist (or could be blockaded). It is easily checked that  $\pi_d(n)$  decreases with  $n$ , meaning that *deterred piracy (under fixed copying costs) is more harmful to producers the larger the number of information goods produced.*

At price  $p_d = (C s_o) / (n s_c)$ , legitimate users achieve a surplus of

$$CL_d(n) = \int_{\frac{C}{n s_c}}^1 n \left( \theta s_o - \frac{C s_o}{n s_c} \right) d\theta = \frac{s_o (n s_c - C)^2}{2 n s_c^2},$$

which is also equal to total consumer surplus since all other consumers prefer not using any information good. It is straightforward to show that (i)  $CL_d(n) > CL_b(n) = ns_o/8$ , and (ii)  $CL_d(n)$  is an increasing function of  $n$ .

Adding total profits to consumer surplus, we derive our measure of social welfare in the short run:

$$W_d(n) = n\pi_d(n) + CL_d(n) = \frac{s_o(n^2s_c^2 - C^2)}{2ns_c^2}.$$

It turns out that the benefits of piracy for the consumers exceeds the losses for the producers. We observe indeed that (i)  $W_d(n) > W_b(n) = 3ns_o/8$ , and (ii)  $W_d(n)$  is an increasing function of  $n$ . We therefore conclude that, as in the variable copying cost case, *deterred piracy improves social welfare in the short-run* (with respect to a hypothetical economy where piracy would be infeasible). The difference with the variable copying cost case is that *welfare improves further as the number of information goods produced increases*.

Because short-run profits decrease with the number of producers, we cannot derive the long-run, free-entry, equilibrium as easily as in the variable-copying cost model.<sup>15</sup> However, we can conjecture that (deterred) piracy is likely to have a more detrimental effect on long-run welfare (because, as more goods get produced, the marginal producer faces both higher fixed creation costs and lower gross profits).

#### 4.4 Accommodated piracy

We now look for a symmetric Bertrand-Nash equilibrium in which producers find it optimal to tolerate piracy. If all originals are priced the same, users will treat all goods alike. That is, the market will be segmented as in the variable-copying cost model: low- $\theta$  users will not use any good, intermediate- $\theta$  users will copy all goods, and high- $\theta$  users will purchase all goods. We need now to determine which common price will achieve such market segmentation.

Suppose that  $(n - 1)$  firms charge a common price  $p$  and that firm  $i$  chooses some price  $p_i$  in the vicinity of  $p$ . To derive the demand facing firm  $i$ , we need to identify the user who is indifferent between buying or copying all goods. This user is identified by  $\tilde{\theta}$  such that  $\tilde{\theta}s_o - p_i + (n - 1)(\tilde{\theta}s_o - p) = n\tilde{\theta}s_c - C$ ; that is

$$\tilde{\theta} = \frac{p_i + (n - 1)p - C}{n(s_o - s_c)}.$$

Because all users with a larger  $\theta$  than  $\tilde{\theta}$  will buy all goods, the demand facing firm  $i$  (as long as  $p_i$  is not too different from  $p$ ) is given by

$$D_i(p_i, p) = 1 - \frac{p_i + (n - 1)p - C}{n(s_o - s_c)}. \quad (15)$$

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<sup>15</sup>Using the same uniform distribution of creation costs as in Section 3, we find now the free-entry number of information goods as the solution to a cubic (instead of linear) equation.

Maximizing  $\pi_i(p_i, p) = D_i(p_i, p)p_i$  over  $p_i$  yields firm  $i$ ' reaction function:

$$R_i(p) = \frac{1}{2} (n(s_o - s_c) - (n-1)p + C).$$

Our candidate for a symmetric Bertrand-Nash equilibrium with accommodated piracy,  $p_a$ , must solve  $p_a = R_i(p_a)$ , which yields

$$p_a = \frac{n(s_o - s_c) + C}{n+1}. \quad (16)$$

Naturally, we need now to investigate under which conditions all firms charging  $p_a$  is indeed a Nash equilibrium. More precisely, supposing that all other firms charge  $p_a$ , we must make sure that firm  $i$  has no incentive to set a price that would bring it to a different segment of demand than (15). To do so, we need to determine exactly what the alternative demand segments look like and when they are observed. As before, the starting point is condition (14), which rewrites here as

$$\begin{aligned} \theta s_o - p_i + R_a &\geq L_a, \\ \text{with } \begin{cases} R_a \equiv \max\{(n-1)(\theta s_o - p_a), (n-1)\theta s_c - C, 0\} \\ L_a \equiv \max\{(n-1)(\theta s_o - p_a), n\theta s_c - C, 0\}. \end{cases} \end{aligned} \quad (17)$$

We start, in the next lemma, by discarding a whole range of cases in which symmetric piracy accommodation *cannot* be a Nash equilibrium.

**Lemma 3** *If  $C \geq C_d$ , then all firms charging  $p_a$  is not a Bertrand-Nash equilibrium.*

**Proof.** See Appendix 6.3. ■

The result of Lemma 3 is not surprising. By analogy with the variable-copying cost model, we expect piracy accommodation to lead to higher prices than piracy deterrence when accommodation is chosen as the most profitable option by the firms. That is, if accommodation is an equilibrium, then  $p_a > p_d$  which is equivalent to  $C < C_d$ . If the opposite is true, copying is costly enough to allow firm  $i$  to behave as an unconstrained monopolist when all other firms charge  $p_a$ .

Lemma 3 provides a necessary condition (i.e.,  $C < C_d$ ) for a symmetric equilibrium with piracy accommodation. However, this condition is far from sufficient: additional conditions have to be met to prevent unilateral deviations. Unfortunately, these conditions are very tedious to derive as they depend on the precise configuration of demand (which depends itself on the values of the parameters, in a much more complicated way than for  $C \geq C_d$ ). As a consequence, we shall not attempt here to give a precise characterization of the configurations of parameters where symmetric piracy accommodation is a Nash



equilibrium. Instead, we will focus on one specific case and use it to illustrate the nature of unilateral deviations from the accommodation price  $p_a$ .

Let us consider situations characterized by the following conditions:

$$\begin{cases} s_c < \frac{n-1}{n} s_o, \\ C < \frac{n^2(n-1)s_c(s_o-s_c)}{(n^2-1)s_o+2ns_c} < C_d. \end{cases} \quad (18)$$

Roughly speaking, we take copies as a relatively poor, but inexpensive, alternative to originals.<sup>16</sup> In such situations, it can be shown that the demand function facing firm  $i$  when all other firms set the accommodation price  $p_a$  is as follows:<sup>17</sup>

$$D_i(p_i, p_a) = \begin{cases} D^0 \equiv 0 & \text{for } p_i \geq s_o, \\ D^1 \equiv 1 - \frac{p_i}{s_o} & \text{for } s_o \frac{(n-1)p_a - C}{(n-1)s_o - ns_c} \leq p_i \leq s_o, \\ D^2 \equiv 1 - \frac{p_i + (n-1)p_a - C}{n(s_o - s_c)} & \text{for } p_a - \frac{C}{n-1} \leq p_i \leq s_o \frac{(n-1)p_a - C}{(n-1)s_o - ns_c}, \\ D^3 \equiv 1 - \frac{p_i}{s_o - s_c} & \text{for } \frac{Cs_o}{ns_c} \leq p_i \leq p_a - \frac{C}{n-1}, \\ D^4 \equiv 1 - \frac{p_i}{s_o - s_c} + \frac{C - p_i}{ns_c - s_o} - \frac{p_i}{s_o} & \text{for } \frac{C(s_o - s_c)}{(n-1)s_o} \leq p_i \leq \frac{Cs_o}{ns_c}, \\ D^5 \equiv 1 - \frac{p_i}{s_o} & \text{for } p_i \leq \frac{C(s_o - s_c)}{(n-1)s_o}. \end{cases} \quad (19)$$

Let us shed some light on the various segments composing the demand function. As soon as the price of original  $i$  is lower than  $s_o$ , some users are willing to purchase the good. In segment  $D^1$ , the price is so high that only the most eager (i.e., high  $\theta$ 's) consumers purchase good  $i$ . These consumers decide whether to purchase good  $i$  or not to use it, given that they purchase anyway all the other, cheaper, information goods.<sup>18</sup> Hence, condition (17) boils down for them to  $\theta s_o \geq p_i$ . Segment  $D^2$ , which corresponds to (15), is obtained when  $p_i$  is set in the vicinity of  $p_a$ . On top of the previous high  $\theta$ 's consumers, firm  $i$  also attracts consumers who prefer purchasing rather than copying all goods. By further decreasing its price, firm  $i$  manages to attract some lower- $\theta$  consumers. These consumers are resolute to copy all other goods no matter what; but if  $p_i$  is sufficiently low, they might prefer the original of good  $i$  to the copy. That is, condition (17) writes for these consumers as:  $\theta s_o - p_i + (n-1)\theta s_c - C \geq n\theta s_c - C \iff \theta(s_o - s_c) \geq p_i$ , which generates segment  $D^3$ . Finally, a further decrease in  $p_i$  attracts the very low- $\theta$  consumers who decide to purchase and use only good  $i$ ; segment  $D^4$  (resp.  $D^5$ ) corresponds to the case where these consumers' second most-preferred option is to copy all goods (resp. not to use any good).

<sup>16</sup>The condition on  $C$  is compatible with our initial requirement that  $C > s_c$  if and only if  $(n^2 - n + 2)ns_c < (n^2 - n - 1)(n-1)s_o$ , which is itself compatible with the other initial requirement that  $ns_c > s_o$  as long as  $n > 2$ .

<sup>17</sup>The demonstration is available upon request from the author.

<sup>18</sup>The other goods are cheaper because  $C < C_d$  implies that  $s_o \frac{(n-1)p_a - C}{(n-1)s_o - ns_c} > p_a$ .

Now, we establish conditions under which  $p_i = p_a$  is *not* firm  $i$ 's best response. We first note that  $p_i = p_a$  is the local optimum corresponding to segment  $D^2$  and yields firm  $i$  a profit of<sup>19</sup>

$$\pi_a(n) = \frac{(n(s_o - s_c) + C)^2}{(n+1)^2 n(s_o - s_c)}.$$

This local optimum, however, might not constitute a global optimum. In particular, suppose that firm  $i$  deviates by setting a price corresponding to segment  $D^3$ . Firm  $i$  would maximize its profits under this demand by setting  $p_i = (s_o - s_c)/2$ . This price is feasible provided that  $(s_o - s_c)/2 \geq Cs_o/(ns_c) \iff C \leq ns_c(s_o - s_c)/(2s_o)$ . Supposing that the latter condition is met (which itself requires that  $n > 3$  to make sure that condition (12) can still be satisfied), firm  $i$ 's maximum profit over segment  $D^3$  is equal to  $\pi^3 = (s_o - s_c)/4$ . Some lines of computations establish that the difference  $\pi_a(n) - \pi^3$  is equivalent in sign to  $4C^2 + 8n(s_o - s_c)C - n(n-1)^2(s_o - s_c)^2$ . This quadratic form in  $C$  admits two real roots, one positive and one negative. The positive root is equal to  $(s_o - s_c)((n+1)\sqrt{n} - 2n)/2$ , which can be shown to be larger than  $ns_c(s_o - s_c)/(2s_o)$  under conditions (18). It follows that the quadratic form is negative everywhere, meaning that  $\pi^3 > \pi_a(n)$  in the situations under review.

We close the discussion by highlighting the intuition behind the previous finding. By setting a price sufficiently lower than  $p_a$ , firm  $i$  reaches segment  $D^3$  and a larger number of users. In particular, it attracts those consumers who are better off copying all goods rather than purchasing them all, but who would be ready to purchase good  $i$  (and still copy all other goods) if it was sufficiently cheaper than the other goods. How cheap good  $i$  should be depends on the cost of the copying technology: if this cost is relatively low, copying one less good is no big sacrifice for the users, and a relatively small discount on good  $i$  will induce them to buy the good. In such instances, the increase in the number of users makes up for the decrease in the unit price and the deviation is profitable.<sup>20</sup>

## 5 Conclusion

Information goods fall in the category of *public goods with exclusion*, that is, “public goods the consumption of which by individuals can be controlled, measured and subjected to payment or other contractual limitation” (Drèze, 1980). Exclusion can be achieved through legal authority and/or technical means.

<sup>19</sup>It can be checked that (i)  $\pi_a(n)$  is a decreasing function of  $n$ , and (ii)  $C < C_d$  implies that  $\pi_a(n) < \pi_b(n) = s_o/4$ .

<sup>20</sup>For higher fixed copying costs—i.e., for  $C > ns_c(s_o - s_c)/(2s_o)$ , firm  $i$ 's best possible deviation is to set the corner solution price  $p_i = Cs_o/(ns_c)$ . Yet, it is still possible that this constrained solution yields a higher profit than  $\pi_a(n)$ .

However, simply specifying intellectual property laws does not ensure that they will be enforced; similarly, technical protective measures are often imperfect and can be “cracked”. As a result, *piracy* (or illicit copying) cannot be completely evacuated. It is therefore extremely important to understand how piracy affects the demand for legitimate information goods and the pricing behavior of their producers. It is equally important, for policy purposes, to identify clearly the welfare implications of piracy.

This paper addresses these questions within a simple, unified model of competition between originals and copies. We use the vertical differentiation framework proposed by Mussa and Rosen (1978): copies are seen as lower-quality alternatives to originals. In a benchmark model, we consider the market for a single information good. We identify conditions about the relative attractiveness of copies under which the producer either can safely ignore the threat of piracy, or has to modify his behavior and decide whether to ‘deter’ or ‘accommodate’ piracy. In the latter two cases, we show that the competition created by piracy enhances social welfare. However, the welfare increase comes at the expense of the producer’s profits, which might then be insufficient to cover the (potentially high) fixed cost of creation.

To account for this traditional trade-off between *ex ante* and *ex post* efficiency considerations, we extend the benchmark model by considering an arbitrary number of information goods. We consider two distinct scenarios. In the first scenario, we assume that the copying technology involves a constant unit cost and no fixed cost. Under this assumption, demands for originals are completely independent of one another and we can simply reproduce the results of the benchmark model. Assuming a fixed creation cost that varies through producers, we derive the free-entry number of information goods. We can then balance *ex ante* and *ex post* efficiency considerations and show that piracy is likely to damage welfare in the long run (unless copies are a poor alternative to originals and/or are expensive to acquire). The tractability of the model allows us to enrich the welfare analysis by introducing, in turn, network externalities and copying using a peer-to-peer technology.

The second scenario assumes that the copying technology involves a positive fixed cost and no marginal cost. The demands for originals are now interdependent because consumers base their decision to copy on the fixed cost of the technology and on the prices of *all* originals. Due to the complexity of the demand system and of the resulting strategic pricing game, we are unable to provide a complete characterization of the set of Bertrand-Nash equilibria. However, we closely examine symmetric equilibria in which piracy is either blockaded, deterred or accommodated. We show, in particular, that the latter two equilibria rely on a set of rather restrictive conditions, as the incentives for unilateral deviation are high: producers tend to free-ride (by setting higher prices) when it comes to deter piracy, or they tend to undercut when it comes

to accommodate piracy.

The directions for future research are twofold. First, and quite obviously, more work needs to be done on the fixed-copying cost model. The characterization of Bertrand-Nash equilibria should be completed. We need not only to characterize all symmetric equilibria in pure strategies, but also to envision asymmetric equilibria and mixed strategies. It is indeed very likely that mixed strategies cannot be dispensed with in this context: because demand functions are discontinuous, payoff functions may fail to be quasi-concave, which may lead to the non-existence of an equilibrium in pure strategies.

The second direction for future research consists in exploiting the two versions of the model to address topical issues. For instance, we could try and assess the effects of enhancing technical protective measures for information goods. A case of interest is the so-called “unrippable” CD: because the technical measure seems to decrease the quality of *both* originals and copies (it is claimed that these CDs cannot be copied but, at the same time, legitimate users might not be able to play the CD on the appliance of their choice), it is not a priori evident that such strategy is profitable.

## 6 Appendix

### 6.1 Proof of Proposition 5

Suppose that  $C \geq (n/2) s_c$  and  $p_j = p_b = s_o/2 \forall j \neq i$ . Let us show that  $p_i = p_b$  is firm  $i$ 's best response in this case. When all other firms charge  $s_o/2$ , condition (14) becomes

$$\begin{aligned} \theta s_o - p_i + R_b &\geq L_b, \\ \text{with } \begin{cases} R_b \equiv \max\{(n-1) \frac{s_o}{2} (2\theta - 1), (n-1) \theta s_c - C, 0\} \\ L_b \equiv \max\{(n-1) \frac{s_o}{2} (2\theta - 1), n\theta s_c - C, 0\}. \end{cases} \end{aligned} \quad (20)$$

(1) We first establish that  $L_b$  cannot be equal to  $(n-1) \theta s_c - C$ . Indeed, suppose the contrary. Then, we must have (i)  $(n-1) \theta s_c - C \geq (n-1) \frac{s_o}{2} (2\theta - 1) \iff \theta \leq [(n-1) (s_o/2) - C] / [(n-1) (s_o - s_c)]$ , and (ii)  $(n-1) \theta s_c - C \geq 0 \iff \theta \geq C / [(n-1) s_c]$ . It is easily checked that the interval on  $\theta$  defined by the latter two inequalities is non-empty provided that  $C < (n-1) s_c/2$ , which violates our initial assumption. We thus conclude that  $L_b$  equals  $(n-1) \frac{s_o}{2} (2\theta - 1)$  for  $\theta \geq 1/2$ , and 0 otherwise.

(2) As for  $R_b$ , we first show that  $R_b = L_b$  when  $(n-1) s_o > n s_c$ . In this case,  $R_b$  cannot be equal to  $n\theta s_c - C$ . Indeed, using a similar argument as above,  $R_b = n\theta s_c - C$  would necessitate that  $\theta$  be comprised between  $C / (n s_c)$  and  $[(n-1) (s_o/2) - C] / [(n-1) s_o - n s_c]$ , which is impossible for  $C \geq (n/2) s_c$ . It follows that  $R_b = L_b$  and that consumers buying original  $i$  are characterized by  $\theta s_o - p_i \geq 0$ . Therefore, firm  $i$  faces a demand  $D_i(p_i, p_b) = (1 - p_i/s_o)$  and its profit-maximizing price is  $p_i = p_b$ , which completes the proof for this case.

(3) When  $(n-1)s_o < ns_c$ ,  $R_b$  can take three values:

$$R_b = \begin{cases} n\theta s_c - C & \text{for } \theta \geq \frac{C-(n-1)(s_o/2)}{ns_c-(n-1)s_o}, \\ (n-1)\frac{s_o}{2}(2\theta-1) & \text{for } \frac{1}{2} \leq \theta \leq \frac{C-(n-1)(s_o/2)}{ns_c-(n-1)s_o}, \\ 0 & \text{for } \theta \leq \frac{1}{2}. \end{cases}$$

Note that, because  $\theta \leq 1$ , the first option ( $R_b = n\theta s_c - C$ ) is possible only if  $C < \bar{C} \equiv ns_c - \frac{n-1}{2}s_o$  (with  $\bar{C} > (n/2)s_c$ ). If the reverse is true, then we have again that  $R_b = L_b$  and the proof is completed. Supposing  $C < ns_c - \frac{n-1}{2}s_o$ , define  $p_{12} \equiv C - ns_c + \frac{n+1}{2}s_o$  and  $p_{23} \equiv s_o \frac{C-(n-1)(s_o/2)}{ns_c-(n-1)s_o}$ . We can summarize the analysis of inequality (20) as follows:

for $\theta \in$	(20) rewrites as	never met if	always met if
$\frac{C-(n-1)(s_o/2)}{ns_c-(n-1)s_o}, 1$	$\theta \geq \frac{p_i+(n-1)(s_o/2)-C}{n(s_o-s_c)}$	$p_i \geq p_{12}$	$p_i \leq p_{23}$
$0, \frac{C-(n-1)(s_o/2)}{ns_c-(n-1)s_o}$	$\theta \geq \frac{p_i}{s_o}$	$p_i \geq p_{23}$	/

We can now write the demand facing firm  $i$ :

$$D_i\left(p_i, \frac{s_o}{2}\right) = \begin{cases} 0 & \text{if } p_i \geq p_{12}, \\ 1 - \frac{p_i+(n-1)(s_o/2)-C}{n(s_o-s_c)} & \text{if } p_{23} \leq p_i \leq p_{12}, \\ 1 - \frac{p_i}{s_o} & \text{if } p_i \leq p_{23}. \end{cases}$$

If firm  $i$  chooses the third segment of demand, the (unconstrained) optimal price is  $p_i = s_o/2$ . It is readily checked that this price meets the constraint when  $C \geq (n/2)s_c$ . This option allows thus the firm to achieve a profit of  $\pi_{(3)} = s_o/4$ . On the other hand, if firm  $i$  chooses the second segment of demand, it can be shown that, with  $C \geq (n/2)s_c$ , the interior optimum is not feasible. The best firm  $i$  can do is to charge  $p_i = p_{23}$ , which yields a profit of

$$\pi_{(2)} = \frac{s_o(2C - (n-1)s_o)(2ns_c - (n-1)s_o - 2C)}{4(ns_c - (n-1)s_o)^2}.$$

Now, because

$$\pi_{(3)} - \pi_{(2)} = \frac{s_o(2C - ns_c)^2}{4(ns_c - (n-1)s_o)^2} > 0,$$

firm  $i$  prefers the third segment of demand, which implies that  $i$ 's best response is still  $p_i = s_o/2$ . *QED*

## 6.2 Proof of Proposition 6

Suppose  $p_j = \bar{p} = (Cs_o)/(ns_c) \forall j \neq i$ . We need to determine when  $p_i = p_d$  is firm  $i$ 's best response. We start by deriving the demand function facing firm  $i$ ,  $D_i(p_i, p_d)$ . When all other firms charge  $p_d$ , condition (14) becomes

$$\begin{aligned} \theta s_o - p_i + R_d &\geq L_d, \\ \text{with } \begin{cases} R_d \equiv \max\{(n-1)(\theta s_o - p_d), (n-1)\theta s_c - C, 0\} \\ L_d \equiv \max\{(n-1)(\theta s_o - p_d), n\theta s_c - C, 0\}. \end{cases} \end{aligned} \quad (21)$$

Straightforward computations establish that the exact values of  $MR$  and  $ML$  are as follows:

$$\begin{aligned}
& * \text{ for } 0 \leq \theta \leq \frac{C}{ns_c}, \quad R_d = L_d = 0, \\
& * \text{ for } \frac{C}{ns_c} \leq \theta \leq 1, \quad R_d = (n-1)(\theta s_o - p_d) \\
& \quad L_d = \begin{cases} (n-1)(\theta s_o - p_d) & \text{if } ns_c \leq (n-1)s_o, \\ n\theta s_c - C & \text{if } ns_c > (n-1)s_o. \end{cases}
\end{aligned}$$

If  $ns_c \leq (n-1)s_o$ , then  $R_d = L_d \forall \theta$ . It follows that condition (21) boils down to  $\theta s_o - p_i \geq 0$ , which implies that  $D_i(p_i, p_d) = 1 - p_i/s_o$ , and that firm  $i$ 's optimal price is  $p_i = s_o/2 \neq p_d$ . Therefore, for symmetric piracy deterrence to be a Nash equilibrium, it is necessary that  $ns_c > (n-1)s_o$ . In such a case, condition (21) has two possible forms:  $\theta s_o - p_i \geq 0$  for  $\theta \leq C/(ns_c)$ , or  $n\theta s_o - p_i - (n-1)p_d \geq n\theta s_c - C$  for  $\theta \geq C/(ns_c)$ . Computing the conditions on  $p_i$  under which each specific form is satisfied in the relevant range, we derive  $i$ 's demand function as

$$D_i(p_i, \bar{p}) = \begin{cases} 0 & \text{if } p_i \geq n(s_o - s_c) + C - (n-1)p_d, \\ 1 - \frac{p_i + (n-1)p_d - C}{n(s_o - s_c)} & \text{if } p_d \leq p_i \leq n(s_o - s_c) + C - (n-1)p_d, \\ 1 - \frac{p_i}{s_o} & \text{if } p_i \leq p_d. \end{cases}$$

For firm  $i$ 's best response to be  $p_i = p_d$ , the interior solutions when  $i$ 's maximizes over the second or third segments of demand must be infeasible. As for the second segment, the unconstrained optimum is  $p_i = p_{(2)} \equiv (1/2)(n(s_o - s_c) + C - (n-1)p_d)$ ; one easily checks that  $p_{(2)} \leq p_d$  for  $C \geq C_d$ . As for the third segment, the unconstrained optimum is  $p_i = s_o/2$ , which is clearly larger than  $p_d$  for  $C \leq (n/2)s_c$ . This completes the proof. *QED*

### 6.3 Proof of Lemma 3

The first thing to note is that  $C \geq C_d \iff p_a \leq (s_o C)/(ns_c)$ . In this case, the comparison between  $R_a$  and  $L_a$  is rather simple. First, if  $(n-1)s_c > ns_o$ , it is easily checked that  $R_a = L_a \forall \theta$ . It follows that condition (17) boils down to  $\theta s_o - p_i \geq 0$ , and we know that firm  $i$ 's best reply is then  $p_i = s_o/2 \neq p_a$ , which establishes our result for this particular case. Second, if  $(n-1)s_c < ns_o$ , the analysis of condition (17) can be summarized by the following table (based on straightforward computations):

for $\theta \in$	(17) rewrites as	never met if	always met if
$\left[ \frac{C - (n-1)p_a}{ns_c - (n-1)s_o}, 1 \right]$	$\theta \geq \frac{p_i + (n-1)p_a - C}{n(s_o - s_c)}$	$p_i \geq 2p_a$	$p_i \leq s_o \frac{C - (n-1)p_a}{ns_c - (n-1)s_o}$
$\left[ 0, \frac{C - (n-1)p_a}{ns_c - (n-1)s_o} \right]$	$\theta \geq \frac{p_i}{s_o}$	$p_i \geq s_o \frac{C - (n-1)p_a}{ns_c - (n-1)s_o}$	/

We can now write the demand facing firm  $i$ :

$$D_i(p_i, p_a) = \begin{cases} 0 & \text{if } p_i \geq 2p_a, \\ 1 - \frac{p_i + (n-1)p_a - C}{n(s_o - s_c)} & \text{if } s_o \frac{C - (n-1)p_a}{ns_c - (n-1)s_o} \leq p_i \leq 2p_a, \\ 1 - \frac{p_i}{s_o} & \text{if } p_i \leq s_o \frac{C - (n-1)p_a}{ns_c - (n-1)s_o}. \end{cases}$$

For  $p_a$  to be firm  $i$ 's best reply, it must be the case that firm  $i$  selects the second segment of demand and that the interior solution to its profit-maximization problem be feasible. This is so provided that

$$p_a \geq s_o \frac{C - (n-1)p_a}{ns_c - (n-1)s_o} \iff p_a \geq \frac{s_o C}{ns_c},$$

which contradicts our initial assumption and completes the proof.

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